A Two-Stage Solution Procedure to Joint Subcarrier Assignment and Global Energy-Efficient Power Allocation in Energy-Harvesting Two-Tier Downlink NOMA HetNets

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***Abstract*—This paper considers the problem of joint subcarrier assignment and global energy-efficient power allocation (J-SA-GEE-PA) for energy-harvesting (EH) two-tier downlink non-orthogonal multiple-access (NOMA)-based heterogeneous networks (HetNets). Particularly, the HetNet consists of a macro base-station (MBS) and a number of small base-stations (SBSs), which are solely powered via renewable-energy sources. The aim is to solve the joint subcarrier assignment and global energy-efficiency problem subject to quality-of-service (QoS) per user and other practical constraints. However, the formulated J-SA-GEE-PA problem happens to be non-convex and NP-hard, and thus is computationally-prohibitive. In turn, the J-SA-GEE-PA problem is split into two sub-problems: (1) subcarrier assignment via many-to-many matching, and (2) GEE-maximizing power allocation. In the first sub-problem, the subcarriers are assigned to users via the deferred acceptance algorithm. As for the second sub-problem, the GEE-PA problem is solved optimally via a low-complexity algorithm. After that, a two-stage solution procedure is devised to efficiently solve the J-SA-GEE-PA problem. Simulation results are presented to validate the proposed solution procedure, where it is shown to efficiently yield comparable network global energy-efficiency to the J-SA-GEE-PA scheme; however, with lower computational-complexity.**

***Index Terms*—Energy-efficiency, heterogeneous networks, non-orthogonal multiple-access, power allocation, subcarrier assignment**

I. Introduction

The ever increasing demand for massive connectivity, spectral- and energy-efficiency, and higher throughput—while meeting quality-of-service (QoS) requirements—has induced the design of effective resource allocation solutions for fifth-generation (5G) cellular networks and beyond. To meet such demands, heterogeneous networks (HetNets) have been proposed as a means for network densification, so as to improve cellular coverage, capacity and spectrum utilization [1]. Also, non-orthogonal multiple-access (NOMA) has been proposed to further improve spectrum utilization and capacity [2]. Moreover, due to the scarcity of energy resources, energy-harvesting (EH) has emerged as a viable solution to energize cellular networks via renewable-energy sources [3]. Thus, there is an urgent need for optimal and energy-efficient transmission schemes in EH NOMA-enabled HetNets.

Recently, several research works have considered resource allocation in NOMA HetNets. For instance, the tradeoff between energy-efficiency (EE) and spectral-efficiency (SE) in downlink NOMA HetNets is studied in [4]. Particularly, the tradeoff is formulated as a multi-objective optimization problem, subject to maximum transmit power and minimum rate requirements, and the joint sub-channel allocation and power allocation problem is solved via an iterative algorithm. In [5], network throughput maximization in downlink NOMA HetNets is considered, where the problem proves to be NP-hard. Thus, a scheduling scheme and an iterative distributed power control algorithm are proposed, and shown to be superior to OMA HetNets and single-tier NOMA networks. In [6], the authors study the problem of joint user association and power control (J-UA-PC) for energy-efficiency maximization in two-tier downlink NOMA HetNets. Particularly, the base-stations are powered by renewable energy sources as well the conventional grid. In turn, a distributed algorithm for optimal user association—for fixed power allocation—is proposed, and then, a J-UA-PC algorithm is devised to further maximize the energy-efficiency, ultimately outperforming OMA-based networks.

In this paper, the problem of joint subcarrier assignment and global energy-efficient power allocation (J-SA-GEE-PA) for EH two-tier downlink NOMA HetNets is studied. In particular, the HetNet consists of a macro base-station (MBS) and a number of small base-stations (SBSs), which are solely powered via renewable-energy sources. The aim is to solve the J-SA-GEE-PA problem; subject to a QoS constraint per user as well as other practical constraints. However, the formulated J-SA-GEE-PA problem happens to be non-convex and NP-hard [7]. Thus, the J-SA-GEE-PA problem is split into two sub-problems: (1) subcarrier assignment via many-to-many matching, and (2) GEE-maximizing power allocation. The first sub-problem is solved via the deferred acceptance (DA) algorithm [8], within polynomial-time complexity. As for the second sub-problem, the global energy-efficient power allocation is solved optimally via a low-complexity algorithm, while incorporating both intra-cell and inter-cell interferences. After that, a two-stage solution procedure is devised to solve the J-SA-GEE-PA problem by solving the subcarrier assignment and GEE-maximizing power allocation in the MBS- and SBS-tiers, while ensuring stability via swap matching. Simulation results are presented to validate the efficacy of the proposed solution procedure, which will be shown to efficiently yield comparable network global energy-efficiency to the J-SA-GEE-PA scheme (solved via a global optimization package); however, with lower computational-complexity.

The rest of this paper is organized as follows. In Section II, the system model is presented, while Section III presents the J-SA-GEE-PA problem formulation. Section IV models the subcarrier assignment as a matching problem, while Section V presents the algorithmic solution of the global energy-efficient power allocation. Section VI presents the swap matching algorithm, while Section VII outlines the proposed solution procedure. Section VIII presents the simulation results, while Section IX draws the conclusions.

II. System Model

A. Network Model

Consider a two-tier downlink HetNet consisting of N users, a macro base-station (MBS) and M small base-stations (SBSs), where downlink transmission is achieved via NOMA. Let the set of all base-stations be denoted $\mathcal{B} = \{BS_0, BS_1, \ldots, BS_m, \ldots, BS_M\}$, where BS_0 denotes the MBS, while BS_m (for $m = 1, 2, \ldots, M$) denotes the m^{th} SBS. The frequency spectrum is split into a set of K subcarriers, denoted $\mathcal{SC} = \{SC_1, \ldots, SC_k, \ldots, SC_K\}$, where SC_k is the k^{th} subcarrier. Also, let $\mathcal{U} = \{\mathcal{U}_0, \ldots, \mathcal{U}_M\}$ be the set of all user subsets in the network, where $\mathcal{U}_0$ is the subset of users associated with the MBS (i.e. BS_0), while $\mathcal{U}_m$ is the subset of users associated with SBS BS_m. The user subsets $\mathcal{U}_0, \ldots, \mathcal{U}_m, \ldots, \mathcal{U}_M$ partition $\mathcal{U}$. That is, $\mathcal{U}_m \cap \mathcal{U}_{m'} = \emptyset$ for $m \neq m'$, and $\bigcup_{m=0}^{M} \mathcal{U}_m = \mathcal{U}$, with $|\mathcal{U}| = N$. The channel between any base-station BS_m and user $U_i \in \mathcal{U}_m$ over subcarrier SC_k follows narrowband Rayleigh fading with zero-mean N_0-variance additive white Gaussian noise (AWGN). In particular, let $h_{m,i}^k \sim \mathcal{CN}\left(0, d_{m,i}^{-\nu}\right)$ be a zero-mean complex Gaussian random variable with variance $d_{m,i}^{-\nu}$, where $d_{m,i}$ is the corresponding distance, while ν is the path-loss exponent.

Now, let $x_{m,i}^k$ be a binary decision variable, defined as

$$x_{m,i}^k = \begin{cases} 1, & \text{if } U_i \in \mathcal{U}_m \text{ is assigned } SC_k, \\ 0, & \text{otherwise.} \end{cases} \quad (1)$$

Also, let $E_{m,i}^k$ denote the transmit energy allocated to user U_i within BS_m over SC_k. Lastly, let $E^{\max}$ be the total transmit energy per time-slot over each subcarrier $SC_k \in \mathcal{SC}$ (i.e. $\sum_{BS_m \in \mathcal{B}} \sum_{U_i \in \mathcal{U}_m} x_{m,i}^k E_{m,i}^k \leq E^{\max}$, $\forall SC_k \in \mathcal{SC}$).

The base-stations are solely powered by renewable-energy sources (e.g. via PV solar panels). Thus, let τ be a unit-duration time-slot[1], in which downlink NOMA transmission occurs. Additionally, let $\mathcal{E}_m^\tau$ be the harvested energy, which is modeled at each BS_m (for $m \in \{0, 1, \ldots, m, \ldots, M\}$) as an independent uniform random variable, as $\mathcal{E}_m^\tau \sim \mathbb{U}\left(0, \mathcal{E}_m^{\max}\right)$, with $\mathcal{E}_m^{\max}$ being the maximum value of harvested energy per time-slot [9]. Hence, the total energy consumption over each time-slot τ at each base-station BS_m must satisfy

$$\sum_{U_i \in \mathcal{U}_m} \sum_{SC_k \in \mathcal{SC}} x_{m,i}^k E_{m,i}^k + E_{C,m} \leq \mathcal{E}_m^\tau, \quad \forall BS_m \in \mathcal{B}, \quad (2)$$

where $E_{C,m}$ is the fixed transceiver energy consumption. Also, let $B_m^{\max}$ be the finite-capacity battery of base-station BS_m, $\forall BS_m \in \mathcal{B}$. Remarkably, any leftover energy from a previous time-slot (i.e. $\tau - 1$) is stored in the battery, and used in the following time-slot (for $\tau \geq 2$), as [9]

$$\mathcal{E}_m^\tau = \min\left(\mathcal{E}_m^\tau + \left(\mathcal{E}_m^{\tau-1} - \left(\sum_{U_i \in \mathcal{U}_m} \sum_{SC_k \in \mathcal{SC}} x_{m,i}^k E_{m,i}^k + E_{C,m}\right)\right), B_m^{\max}\right). \quad (3)$$

Remark 1. The superscript τ for each time-slot is dropped from this point onwards, while implicitly taking into account battery dynamics during network operation.

[1]In turn, the terms "energy" and "power" can be used interchangeably.

B. Transmission Model

Each base-station transmits a superimposed signal of the data symbols of the users associated with it. Thus, the received signal at user U_i of BS_m over SC_k is expressed as

$$y_{m,i}^k = \sqrt{E_{m,i}^k} h_{m,i}^k s_{m,i}^k + I_{m,i}^k + J_{m,i}^k + n_{m,i}^k, \quad (4)$$

where $s_{m,i}^k$ is the signal of $U_i \in \mathcal{U}_m$, such that $\mathbb{E}\left[|s_{m,i}^k|^2\right] = 1$, $\forall U_i \in \mathcal{U}_m$ and $\forall BS_m \in \mathcal{B}$. Also, $n_{m,i}^k$ is the received AWGN sample, while $I_{m,i}^k$ is the intra-cell interference, as given by

$$I_{m,i}^k = h_{m,i}^k \sum_{\substack{U_{i'} \in \mathcal{U}_m \\ i' \neq i}} x_{m,i'}^k \sqrt{E_{m,i'}^k} s_{m,i'}^k. \quad (5)$$

Also, $J_{m,i}^k$ is the inter-cell interference, written as

$$J_{m,i}^k = \sum_{\substack{BS_{m'} \in \mathcal{B} \\ m' \neq m}} h_{m',i}^k \left(\sum_{U_{i'} \in \mathcal{U}_{m'}} x_{m',i'}^k \sqrt{E_{m',i'}^k} s_{m',i'}^k\right). \quad (6)$$

Without loss of generality, let the users in $\mathcal{U}_m$ be ordered in a ascending order according to their *interference channel gains*, as $\tilde{h}_{m,1}^k \leq \cdots \leq \tilde{h}_{m,i}^k \leq \cdots \leq \tilde{h}_{m,|\mathcal{U}_m|}^k$ [6], where

$$\tilde{h}_{m,i}^k \triangleq \frac{|h_{m,i}^k|^2}{\mathcal{J}_{m,i}^k + N_0}, \quad (7)$$

and $\mathcal{J}_{m,i}^k$ is inter-cell interference power, as given by

$$\mathcal{J}_{m,i}^k = \sum_{\substack{BS_{m'} \in \mathcal{B} \\ m' \neq m}} |h_{m',i}^k|^2 \left(\sum_{U_{i'} \in \mathcal{U}_{m'}} x_{m',i'}^k E_{m',i'}^k\right). \quad (8)$$

Based on the principle of NOMA, the set of users $\mathcal{U}_m$ is ordered as $E_{m,1}^k \geq \cdots \geq E_{m,i}^k \geq \cdots \geq E_{m,|\mathcal{U}_m|}^k$. Assuming perfect SIC, the received SINR of $U_i \in \mathcal{U}_m$ over SC_k is

$$\gamma_{m,i}^k = \frac{x_{m,i}^k E_{m,i}^k |h_{m,i}^k|^2}{\mathcal{I}_{m,i}^k + \mathcal{J}_{m,i}^k + N_0}, \quad (9)$$

where $\mathcal{I}_{m,i}^k$ is the intra-cell interference power after SIC, as

$$\mathcal{I}_{m,i}^k = |h_{m,i}^k|^2 \left(\sum_{\substack{U_{i'} \in \mathcal{U}_m \\ i' > i}} x_{m,i'}^k E_{m,i'}^k\right). \quad (10)$$

Thus, the achievable rate of user $U_i \in \mathcal{U}_m$ over subcarrier $SC_k \in \mathcal{SC}$ is expressed as

$$R_{m,i}^k\left(\mathbf{E}^k, \mathbf{x}^k\right) = \log_2\left(1 + \gamma_{m,i}^k\left(\mathbf{E}^k, \mathbf{x}^k\right)\right), \quad (11)$$

where $\mathbf{x}^k \triangleq \left[x_{m,i}^k\right]$ is the network subcarrier assignment matrix over subcarrier SC_k. In addition, $\mathbf{E}^k \triangleq \left[E_{m,i}^k\right]$ is the network transmit energy matrix over subcarrier SC_k. Hence, the total achievable rate of user $U_i \in \mathcal{U}_m$ is obtained as

$$R_{m,i}\left(\mathbf{E}, \mathbf{x}\right) = \sum_{SC_k \in \mathcal{SC}} R_{m,i}^k\left(\mathbf{E}^k, \mathbf{x}^k\right), \quad (12)$$

where $\mathbf{x}$ is the network subcarrier assignment matrix; while $\mathbf{E}$ is the network transmit energy matrix. To guarantee QoS, all network users in each tier must satisfy minimum rate requirements, as $R_{m,i}\left(\mathbf{E}, \mathbf{x}\right) \geq \mathbb{R}_{\min}$, where $\mathbb{R}_{\min} = \mathbb{R}_{\min}^M$, $\forall U_i \in \mathcal{U}_0$, and $\mathbb{R}_{\min} = \mathbb{R}_{\min}^S$, $\forall U_i \in \mathcal{U}_m, \forall m \geq 1$, with $\mathbb{R}_{\min}^M$ $\left(\mathbb{R}_{\min}^S\right)$ being the minimum rate requirement per MBS (SBS) user.

The network sum-rate is determined as

$$R_T(\mathbf{E},\mathbf{x}) = \sum_{BS_m \in \mathcal{B}} \left(\sum_{U_i \in \mathcal{U}_m} R_{m,i}(\mathbf{E},\mathbf{x}) \right), \quad (13)$$

while the network total energy consumption is given by

$$E_T(\mathbf{E},\mathbf{x}) = \sum_{BS_m \in \mathcal{B}} \left(\sum_{U_i \in \mathcal{U}_m} \sum_{SC_k \in \mathcal{SC}} x_{m,i}^k E_{m,i}^k + E_{C,m} \right). \quad (14)$$

Remark 2. In this work, a frequency reuse factor of one is assumed, where all base-stations can transmit over all the subcarriers, and all users can utilize any of the K subcarriers.

Remark 3. Each MBS user $U_i \in \mathcal{U}_0$ can be paired to at most ζ_i^M subcarriers (i.e. $\sum_{SC_k \in \mathcal{SC}} x_{0,i}^k \leq \zeta_i^M$). Similarly, each SBS user $U_i \in \mathcal{U}_m$ can be assigned to at most ζ_i^S subcarriers (i.e. $\sum_{SC_k \in \mathcal{SC}} x_{m,i}^k \leq \zeta_i^S$, $\forall U_i \in \mathcal{U}_m$, $\forall m \geq 1$).

Remark 4. To reduce SIC complexity and interference, the number of users that can be multiplexed over a subcarrier SC_k in the MBS-tier is constrained to ξ_k^M, while those in the SBS-tier to ξ_k^S. In other words, $\sum_{U_i \in \mathcal{U}_0} x_{0,i}^k \leq \xi_k^M$, and $\sum_{\substack{BS_m \in \mathcal{B}\\ m \geq 1}} \sum_{U_i \in \mathcal{U}_0} x_{m,i}^k \leq \xi_k^S$, $\forall SC_k \in \mathcal{SC}$.

The global energy-efficiency (GEE) is defined as

$$\mathbf{GEE}(\mathbf{E},\mathbf{x}) \triangleq \frac{R_T(\mathbf{E},\mathbf{x})}{E_T(\mathbf{E},\mathbf{x})}. \quad (15)$$

III. Joint Subcarrier Assignment and Global Energy-Efficient Power Allocation

The joint subcarrier assignment and global energy-efficient power allocation (J-SA-GEE-PA) problem is formulated as

J-SA-GEE-PA:

$$\max \quad \mathbf{GEE}(\mathbf{E},\mathbf{x})$$

$$\text{s.t.} \sum_{BS_m \in \mathcal{B}} \sum_{U_i \in \mathcal{U}_m} x_{m,i}^k E_{m,i}^k \leq E^{\max},\ \forall SC_k \in \mathcal{SC}, \quad (16a)$$

$$\sum_{SC_k \in \mathcal{SC}} x_{0,i}^k \leq \zeta_i^M,\ \forall U_i \in \mathcal{U}_0, \quad (16b)$$

$$\sum_{SC_k \in \mathcal{SC}} x_{m,i}^k \leq \zeta_i^S,\ \forall U_i \in \mathcal{U}_m, \forall m \geq 1, \quad (16c)$$

$$\sum_{U_i \in \mathcal{U}_0} x_{0,i}^k \leq \xi_k^M,\ \forall SC_k \in \mathcal{SC}, \quad (16d)$$

$$\sum_{BS_m \in \mathcal{B}} \sum_{U_i \in \mathcal{U}_m} x_{m,i}^k \leq \xi_k^S,\ \forall SC_k \in \mathcal{SC}, \quad (16e)$$

$$R_{0,i}(\mathbf{E},\mathbf{x}) \geq \mathbb{R}_{\text{min}}^M,\ \forall U_i \in \mathcal{U}_0, \quad (16f)$$

$$R_{m,i}(\mathbf{E},\mathbf{x}) \geq \mathbb{R}_{\text{min}}^S,\ \forall U_i \in \mathcal{U}_m, \forall m \geq 1, \quad (16g)$$

$$E_{m,1}^k \geq \cdots \geq E_{m,i}^k \geq \cdots \geq E_{m,|\mathcal{U}_m|}^k,\quad \forall SC_k \in \mathcal{SC}, \forall BS_m \in \mathcal{B}, \quad (16h)$$

$$\sum_{U_i \in \mathcal{U}_m} \sum_{SC_k \in \mathcal{SC}} x_{m,i}^k E_{m,i}^k + E_{C,m} \leq \mathcal{E}_m,\ \forall BS_m \in \mathcal{B}, \quad (16i)$$

$$0 \leq E_{m,i}^k \leq x_{m,i}^k E^{\max},\ \forall SC_k \in \mathcal{SC},\quad \forall U_i \in \mathcal{U}_m, \forall BS_m \in \mathcal{B}, \quad (16j)$$

$$x_{m,i}^k \in \{0,1\},\quad \forall SC_k \in \mathcal{SC}, \forall U_i \in \mathcal{U}_m, \forall BS_m \in \mathcal{B}. \quad (16k)$$

Constraint (16a) ensures that the sum of transmit energy over each $SC_k \in \mathcal{SC}$ does not exceed $E^{\max}$. Constraints (16b) and (16c) ensure that no user in the MBS-tier (SBS-tier) is assigned more than ζ_i^M $\left(\zeta_i^S\right)$ subcarriers. Constraints (16d) and (16e) ensure that the number of MBS (SBS) users multiplexed over any $SC_k \in \mathcal{SC}$ does not exceed ξ_k^M (ξ_k^S). Constraints (16f) and (16g) enforce the minimum rate requirement per MBS and SBS user, respectively, whereas Constraint (16h) enforces the SIC decoding order over each subcarrier. Constraint (16i) ensures that the total energy consumption of each base-station does not exceed the harvested energy. Constraint (16j) ensures that if a user is assigned to a subcarrier (i.e. $x_{m,i}^k = 1$), then its transmit energy $E_{m,i}^k$ does not exceed $E^{\max}$; otherwise, $E_{m,i}^k = 0$. The last constraint defines the values the binary decision variables take.

Remark 5. Problem **J-SA-GEE-PA** is mixed-integer non-linear fractional programming problem, which can be classified as a mixed-integer non-linear programming (MINLP) problem, and thus is non-convex and NP-hard [7].

Based on **Remark 5**, solving problem **J-SA-GEE-PA** is computationally-prohibitive. Alternatively, problem **J-SA-GEE-PA** can be solved by decoupling it into a two sub-problems: (1) subcarrier assignment via many-to-many matching, and (2) GEE-maximizing power allocation, which are solved over two stages (i.e. in each tier).

IV. Subcarrier Assignment via Many-to-Many Stable Matching

The subcarrier assignment sub-problem is modeled as a two-sided many-to-many firms-workers matching problem [10,11]. Specifically, the firms represent the set of subcarriers $\mathcal{SC}$, while the workers resemble the set of users $\mathcal{U}$. Particularly, the goal is to determine the preference lists of the users and subcarriers over each other, and then a stable matching algorithm—based on the Gale-Shapley deferred acceptance (DA) mechanism [8]—is executed to solve the subcarrier assignment problem in each tier (i.e. the users in the MBS-tier, and then the users in the SBS-tier).

A. Definitions

Definition 1 (Preference Relations). Each $U_i \in \mathcal{U}$ has a strict and transitive preference relation $\succ_U$ over the set $\mathcal{SC} \cup \{\emptyset\}$, while each $SC_k \in \mathcal{SC}$ has a strict and transitive preference relation $\succ_{SC_k}$ over $\mathcal{U} \cup \{\emptyset\}$, where $\emptyset$ denotes the possibility of a user (or subcarrier) remaining unassigned.

Definition 2 (Acceptability). A subset of MBS users $\overline{\mathcal{U}}_0 \subseteq \mathcal{U}_0$ is said to be acceptable to subcarrier $SC_k \in \mathcal{SC}$ if and only if (iff) $\sum_{U_i \in \overline{\mathcal{U}}_0} E_{0,i}^k \leq E^{\max}$. Similarly, and given the assigned MBS users, a subset of SBS users $\overline{\mathcal{U}}_m \subseteq \mathcal{U}_m$ within each SBS ($\forall m \geq 1$) is deemed acceptable to subcarrier SC_k iff $\sum_{U_i \in \overline{\mathcal{U}}_0} E_{0,i}^k + \sum_{\substack{BS_m \in \mathcal{B}\\ m \geq 1}} \sum_{U_i \in \overline{\mathcal{U}}_m} E_{m,i}^k \leq E^{\max}$. Also, a subset of subcarriers is said to be acceptable to a MBS user $U_i \in \mathcal{U}_0$ iff $R_{0,i}(\mathbf{E},\mathbf{x}) \geq \mathbb{R}_{\text{min}}^M$ over that subset. In a similar fashion, a subset of subcarriers is deemed acceptable to a SBS user $U_i \in \mathcal{U}_m$ (for $m \geq 1$) iff $R_{m,i}(\mathbf{E},\mathbf{x}) \geq \mathbb{R}_{\text{min}}^S$.

Definition 3 (Preference Lists). Let $\mathcal{P}_{U_i}$ be the preference list of $U_i \in \mathcal{U}_m$ ($\forall BS_m \in \mathcal{B}$), which contains the subsets of acceptance subcarriers in descending order. Similarly, let $\mathcal{P}_{SC_k}$ be the preference list of $SC_k \in \mathcal{SC}$, where the acceptable subsets of users are ordered in a descending manner.

Intuitively, the higher the achievable rate of each user over a subset of subcarriers is, the more preferred that subset is. Contrarily, the lower the sum of transmit energy of a subset of users over a subcarrier is, the more preferred that subset is.

Definition 4 (Matching-MBS). A matching in the MBS-tier is a mapping $\mathcal{M}^M \in \mathcal{U}_0 \times \mathcal{SC}$, such that[2]:

(1) $U_i \in \mathcal{M}^M(SC_k)$ iff $SC_k \in \mathcal{M}^M(U_i)$, $\forall SC_k \in \mathcal{SC}$, $\forall U_i \in \mathcal{U}_0$.
(2) $\left|\mathcal{M}^M(U_i)\right| \leq \zeta_i^M$, $\forall U_i \in \mathcal{U}_0$.
(3) $\left|\mathcal{M}^M(SC_k)\right| \leq \xi_k^M$, $\forall SC_k \in \mathcal{SC}$.

Definition 5 (Matching-SBS). A matching in the SBS-tier is a mapping $\mathcal{M}^S \in \overline{\mathcal{U}} \times \mathcal{SC}$, such that[3]:

(1) $U_i \in \mathcal{M}^S(SC_k)$ iff $SC_k \in \mathcal{M}^S(U_i)$, $\forall SC_k \in \mathcal{SC}$, $\forall U_i \in \mathcal{U}_m$, and $\forall m \geq 1$.
(2) $\left|\mathcal{M}^S(U_i)\right| \leq \zeta_i^S$, $\forall U_i \in \mathcal{U}_m$, $\forall m \geq 1$.
(3) $\left|\mathcal{M}^S(SC_k)\right| \leq \xi_k^S$, $\forall SC_k \in \mathcal{SC}$.

Definition 6 (Pair Blocking). A matching $\mathcal{M}$ is said to be blocked by a pair (U_i, SC_k) if they are not assigned under $\mathcal{M}$, and SC_k finds U_i acceptable iff U_i finds SC_k acceptable, $|\mathcal{M}(U_i)| < \zeta_i$, or $SC_k \succ_{U_i} SC_l$, for a $SC_l \in \mathcal{M}(U_i)$, or $|\mathcal{M}(SC_k)| < \xi_k$, or $U_i \succ_{SC_k} U_j$, for a $U_j \in \mathcal{M}(SC_k)$.

Definition 7 (Stability). A matching $\overline{\mathcal{M}}$ is said to be stable if is not blocked by any (user, subcarrier) pair [12].

B. Algorithm Description

In the DA algorithm, each $U_i \in \mathcal{U}_m$ proposes to its most preferred ζ_i subcarriers. Then, each $SC_k \in \mathcal{SC}$ places the ξ_k most preferred users on its waiting list, and rejects the rest. Any rejected U_i in the previous step proposes again to its most preferred ζ_i subcarriers, provided that none of the proposed-to subcarriers have previously rejected it. After that, each SC_k selects the best ξ_k users from among the newly proposing users and those already on its waiting list, updates its waiting list, and rejects the rest. This process repeats until convergence to the stable matching $\overline{\mathcal{M}}$, as outlined in Algorithm 1.

Remark 6. The **DA** algorithm is applicable to the MBS- and SBS-tiers. For the MBS-tier, the preference lists of all users associated with the MBS (i.e. $U_i \in \mathcal{U}_0$) are utilized to obtain the MBS stable matching solution $\overline{\mathcal{M}}^M$. As for the SBS-tier, the preference lists of all users within all SBSs (i.e. excluding the MBS users, and hence $\overline{\mathcal{U}} = \mathcal{U} \setminus \mathcal{U}_0$) are utilized to obtain the SBS stable matching solution (i.e. $\overline{\mathcal{M}}^S$).

C. Properties

1) Existence and Stability: Every instance of the **DA** algorithm yields at least one stable matching solution [13].

2) Convergence: The **DA** algorithm converges in a finite number of iterations to a stable matching solution $\overline{\mathcal{M}}$ [8].

3) Complexity: The **DA** algorithm has a polynomial-time complexity of $\mathcal{O}(L^2)$, where $L = \max(N, K)$, with N and K being the number of users and subcarriers, respectively [8]. Specifically, for the MBS-tier, $L = \max(|\mathcal{U}_0|, K)$, while for the SBS-tier, $L = \max\left(|\overline{\mathcal{U}}|, K\right)$.

[2] $\mathcal{M}(U_i)$ denotes the subset of subcarriers assigned to user $U_i \in \mathcal{U}_m$, while $\mathcal{M}(SC_k)$ denotes the subset of users assigned to subcarrier SC_k.

[3] Note that $\overline{\mathcal{U}} = \mathcal{U} \setminus \mathcal{U}_0$ (or equivalently $\overline{\mathcal{U}} = \bigcup_{m=1}^{M} \mathcal{U}_m$), which represents the subset of all SBS users.

Algorithm 1: Deferred Acceptance (DA)

Input: Preference lists $\mathcal{P}_{U_i}$ and $\mathcal{P}_{SC_k}$, $\forall SC_k \in \mathcal{SC}$, $\forall U_i \in \mathcal{U}_m$.
1: Each $U_i \in \mathcal{U}_m$ proposes to its most preferred ζ_i SCs in $\mathcal{SC}$;
2: Each $SC_k \in \mathcal{SC}$ places on its waiting list the ξ_k most preferred users, and rejects the rest;
3: WHILE (there exists a previously rejected $U_i \in \mathcal{U}_m$ with a non-empty $\mathcal{P}_{U_i}$ that contains at least one SC that has not rejected it before)
4: Any $U_i \in \mathcal{U}_m$ previously rejected by any SC proposes to its most preferred ζ_i SCs, which have not previously rejected it;
5: Each $SC_k \in \mathcal{SC}$ selects the best ξ_k users from the newly proposing users and those on its waiting list, places them on its waiting list, and rejects the rest;
6: END WHILE
Output: Stable matching solution $\overline{\mathcal{M}}$.

V. Global Energy-Efficient Power Allocation

A. Stage 1

After executing the **DA** algorithm, the subcarriers are assigned to the MBS users only. Let $\bar{\mathbf{x}}^M = \left[\bar{x}_{0,i}^k\right]$ (for $\bar{x}_{0,i}^k \in \{0,1\}$) be the subcarrier assignment of the MBS users, where $\bar{\mathbf{x}}^M = \overline{\mathcal{M}}^M$. Thus, the global energy-efficient power allocation in Stage 1 (GEE-PA-Stage-1) for *fixed* MBS users' subcarrier assignment is formulated as

GEE-PA-Stage-1:

$$\max \quad \textbf{GEE}\left(\mathbf{E}^M, \bar{\mathbf{x}}^M\right)$$

$$\text{s.t.} \sum_{U_i \in \mathcal{U}_0} \bar{x}_{0,i}^k E_{0,i}^k \leq E^{\max}, \forall SC_k \in \mathcal{SC}, \tag{17a}$$

$$R_{0,i}\left(\mathbf{E}^M, \bar{\mathbf{x}}^M\right) \geq \mathbb{R}_{\min}^M, \forall U_i \in \mathcal{U}_0, \tag{17b}$$

$$E_{0,1}^k \geq \cdots \geq E_{0,i}^k \geq \cdots \geq E_{0,|\mathcal{U}_0|}^k, \forall SC_k \in \mathcal{SC}, \tag{17c}$$

$$\sum_{U_i \in \mathcal{U}_0} \sum_{SC_k \in \mathcal{SC}} \bar{x}_{0,i}^k E_{0,i}^k + E_{C,0} \leq \mathcal{E}_0, \tag{17d}$$

$$0 \leq E_{0,i}^k \leq \bar{x}_{0,i}^k E^{\max}, \forall U_i \in \mathcal{U}_0, \forall SC_k \in \mathcal{SC}, \tag{17e}$$

where

$$\begin{aligned}\textbf{GEE}\left(\mathbf{E}^M, \bar{\mathbf{x}}^M\right) &= \frac{\sum_{U_i \in \mathcal{U}_0} \sum_{SC_k \in \mathcal{SC}} \log_2\left(1 + \frac{\bar{x}_{0,i}^k E_{0,i}^k |h_{0,i}^k|^2}{\mathcal{I}_{0,i}^k + N_0}\right)}{\sum_{U_i \in \mathcal{U}_0} \sum_{SC_k \in \mathcal{SC}} \bar{x}_{0,i}^k E_{0,i}^k + E_{C,0}} \\ &= \frac{R_T\left(\mathbf{E}^M, \bar{\mathbf{x}}^M\right)}{E_T\left(\mathbf{E}^M, \bar{\mathbf{x}}^M\right)},\end{aligned} \tag{18}$$

and $\mathbf{E}^M = \left[E_{0,i}^k\right]$ is the transmit energy matrix of MBS users.

Remark 7. It can be verified that the numerator of $\textbf{GEE}\left(\mathbf{E}^M, \bar{\mathbf{x}}^M\right)$ is not jointly concave in $\mathbf{E}^M$, and thus problem **GEE-PA-Stage-1** is not convex [14,15].

To efficiently solve problem **GEE-PA-Stage-1**, let us consider the inequality [16]

$$\log_2(1+\gamma) \geq \alpha \log_2(\gamma) + \beta, \tag{19}$$

where $\alpha \triangleq \frac{\bar{\gamma}}{\bar{\gamma}+1}$, and $\beta \triangleq \log_2(1+\bar{\gamma}) - \alpha \log_2(\bar{\gamma})$, with γ, $\bar{\gamma} > 0$, and the inequality is tight for $\gamma = \bar{\gamma}$. Now, by using the variable substitution $E_{0,i}^k = 2^{Q_{0,i}^k}$, the rate function $R_{0,i}\left(\mathbf{E}^M, \bar{\mathbf{x}}^M\right)$ can be re-written as [14]

$$R_{0,i}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)\geq\bar{R}_{0,i}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)\triangleq\sum_{SC_k\in\mathcal{SC}}\alpha_{0,i}^k\left(Q_{0,i}^k+\log_2\left(\bar{x}_{0,i}^k|h_{0,i}^k|^2\right)-\log_2\left(|h_{0,i}^k|^2\left(\sum_{\substack{U_{i'}\in\mathcal{U}_0\\ i'>i}}\bar{x}_{0,i'}^k2^{Q_{0,i'}^k}\right)+N_0\right)\right)+\beta_{0,i}^k. \tag{21}$$

$$\mathbf{GEE}\left(\mathbf{E}^M,\bar{\mathbf{x}}^M\right)\geq\overline{\mathbf{GEE}}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)\triangleq\frac{\sum_{U_i\in\mathcal{U}_0}\bar{R}_{0,i}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)}{\sum_{U_i\in\mathcal{U}_0}\sum_{SC_k\in\mathcal{SC}}\bar{x}_{0,i}^k2^{Q_{0,i}^k}+E_{C,0}}\triangleq\frac{\bar{R}_T\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)}{\bar{E}_T\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)}. \tag{22}$$

$$R_{0,i}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)$$
$$=\sum_{SC_k\in\mathcal{SC}}\log_2\left(1+\frac{\bar{x}_{0,i}^k2^{Q_{0,i}^k}|h_{0,i}^k|^2}{|h_{0,i}^k|^2\left(\sum_{\substack{U_{i'}\in\mathcal{U}_0\\ i'>i}}\bar{x}_{0,i'}^k2^{Q_{0,i'}^k}\right)+N_0}\right), \tag{20}$$

where $\mathbf{Q}^M\triangleq\left[Q_{0,i}^k\right]$ is the transformed transmit energy matrix. Now, using the inequality in (19), $R_{0,i}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)$ in (20) can be lower-bounded as given in (21).

Remark 8. The negative *log-sum-exp* term

$$-\log_2\left(|h_{0,i}^k|^2\left(\sum_{\substack{U_{i'}\in\mathcal{U}_0\\ i'>i}}\bar{x}_{0,i'}^k2^{Q_{0,i'}^k}\right)+N_0\right)$$

is concave [14], and thus the lower-bounded rate function $\bar{R}_{0,i}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)$ is also concave in $\mathbf{Q}^M$, for fixed $\bar{\mathbf{x}}^M$ [17].

Based on the above, the $\mathbf{GEE}\left(\mathbf{E},\bar{\mathbf{x}}\right)$ objective function of problem **GEE-PA-Stage-1** is lower-bounded and re-written as given in (22). Similar transformation applies to the remaining constraints in problem **GEE-PA-Stage-1**. Hence, problem **GEE-PA-Stage-1** can be *reformulated* as

R-GEE-PA-Stage-1:

$$\max\quad\overline{\mathbf{GEE}}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)$$

$$\text{s.t.}\ \sum_{U_i\in\mathcal{U}_0}\bar{x}_{0,i}^k2^{Q_{0,i}^k}\leq E^{\max},\ \forall SC_k\in\mathcal{SC}, \tag{23a}$$
$$\bar{R}_{0,i}\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)\geq\mathbb{R}_{\min}^M,\ \forall U_i\in\mathcal{U}_0, \tag{23b}$$
$$Q_{0,1}^k\geq\cdots\geq Q_{0,i}^k\geq\cdots\geq Q_{0,|\mathcal{U}_0|}^k,\ \forall SC_k\in\mathcal{SC}, \tag{23c}$$
$$\sum_{U_i\in\mathcal{U}_0}\sum_{SC_k\in\mathcal{SC}}\bar{x}_{0,i}^k2^{Q_{0,i}^k}+E_{C,0}\leq\mathcal{E}_0, \tag{23d}$$
$$0\leq2^{Q_{0,i}^k}\leq\bar{x}_{0,i}^kE^{\max},\ \forall U_i\in\mathcal{U}_0,\forall SC_k\in\mathcal{SC}. \tag{23e}$$

Remark 9. All the constraints in Problem **R-GEE-PA-Stage-1** can be verified to be convex and/or linear. Moreover, the objective function is ratio of the concave $\bar{R}_T\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)$ function to the convex $E_T\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)$ function.

Consequently, problem **R-GEE-PA-Stage-1** can be globally optimally solved via Dinkelbach's algorithm [18]. To this end, define the auxiliary function [19][4]

$$\bar{F}\left(\lambda\right)=\max_{\mathbf{Q}^M}\left\{\bar{R}_T\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)-\lambda\bar{E}_T\left(\mathbf{Q}^M,\bar{\mathbf{x}}^M\right)\right\}, \tag{24}$$

where $\lambda\geq0$ is the unique maximizer of $\bar{F}\left(\lambda\right)$, for fixed values of $\alpha_{0,i}^k$ and $\beta_{0,i}^k$, $\forall SC_k\in\mathcal{SC}$, and $\forall U_i\in\mathcal{U}_0$ [20]. The Dinkelbach's algorithm is given in Algorithm 2.

Algorithm 2: Dinkelbach's Algorithm for Solving Problem **R-GEE-PA-Stage-1**

Set $\epsilon\in(0,1)$, $l=0$, and $\lambda^{(0)}=0$.
1: WHILE $\bar{F}\left(\lambda^{(l)}\right)>\epsilon$
2: $\quad\tilde{\mathbf{Q}}^{(l)}=\arg\max_{\mathbf{Q}}\left\{\bar{R}_T\left(\mathbf{Q},\bar{\mathbf{x}}^M\right)-\lambda^{(l)}\bar{E}_T\left(\mathbf{Q},\bar{\mathbf{x}}^M\right)\right\}$;
3: $\quad\bar{F}\left(\lambda^{(l)}\right)=\bar{R}_T\left(\tilde{\mathbf{Q}}^{(l)},\bar{\mathbf{x}}^M\right)-\lambda^{(l)}\bar{E}_T\left(\tilde{\mathbf{Q}}^{(l)},\bar{\mathbf{x}}^M\right)$;
4: $\quad\lambda^{(l+1)}=\frac{\bar{R}_T\left(\tilde{\mathbf{Q}}^{(l)},\bar{\mathbf{x}}^M\right)}{\bar{E}_T\left(\tilde{\mathbf{Q}}^{(l)},\bar{\mathbf{x}}^M\right)}$;
5: $\quad l=l+1$;
6: END WHILE
Output: $\hat{\mathbf{Q}}^M\triangleq\tilde{\mathbf{Q}}^*$.

Proposition 1. Algorithm 2 monotonically improves the values of $\left\{\lambda^{(l)}\right\}_l$, ultimately converging to the global optimal solution $\hat{\mathbf{Q}}^M$ of problem **R-GEE-PA-Stage-1** [21][5].

Based on **Algorithm 2**[6], the obtained solution $\hat{\mathbf{Q}}^M$ must be converted into its original form as $\hat{\mathbf{E}}^M=2^{\hat{\mathbf{Q}}^M}$. Hence, the value of the objective function in (18)–for fixed values of $\alpha_{0,i}^k$ and $\beta_{0,i}^k$—is obtained as

$$\widehat{\mathbf{GEE}}\left(\hat{\mathbf{E}}^M,\bar{\mathbf{x}}^M\right)\triangleq\frac{\sum_{U_i\in\mathcal{U}_0}R_{0,i}\left(\hat{\mathbf{E}}^M,\bar{\mathbf{x}}^M\right)}{\sum_{U_i\in\mathcal{U}_0}\sum_{SC_k\in\mathcal{SC}}\bar{x}_{0,i}^k\hat{E}_{0,i}^k+E_{C,0}}. \tag{25}$$

Recall that the objective function value obtained via **Algorithm 2** is a lower-bound, and thus, it must be improved by repeatedly updating $\alpha_{0,i}^k$ and $\beta_{0,i}^k$, $\forall SC_k\in\mathcal{SC}$, and $\forall U_i\in\mathcal{U}_0$. This can be achieved via Algorithm 3, ultimately yielding the transmit energy matrix $\bar{\mathbf{E}}^M$.

Algorithm 3: Solution of Problem **GEE-PA-Stage-1**

Set $\epsilon\in(0,1)$, $l=0$, find a feasible $\hat{\mathbf{E}}^{(0)}$, and set $\alpha_{0,i}^{k,(0)}=1$ and $\beta_{0,i}^{k,(0)}=0$, $\forall SC_k\in\mathcal{SC}$, and $\forall U_i\in\mathcal{U}_0$.
1: WHILE $\left|\widehat{\mathbf{GEE}}\left(\hat{\mathbf{E}}^{(l+1)},\bar{\mathbf{x}}^M\right)-\widehat{\mathbf{GEE}}\left(\hat{\mathbf{E}}^{(l)},\bar{\mathbf{x}}^M\right)\right|>\epsilon$
2: $\quad l=l+1$;
3: $\quad$Evaluate $\widehat{\mathbf{GEE}}\left(\hat{\mathbf{E}}^{(l)},\bar{\mathbf{x}}^M\right)$;
4: $\quad$Update $\alpha_{0,i}^{k,(l)}$ and $\beta_{0,i}^{k,(l)}$, $\forall SC_k\in\mathcal{SC}$, and $\forall U_i\in\mathcal{U}_0$;
5: $\quad$Determine $\hat{\mathbf{Q}}^{(l)}$ by solving **R-GEE-PA-Stage-1** via **Algorithm 2**;
6: $\quad$Set $\hat{\mathbf{E}}^{(l)}=2^{\hat{\mathbf{Q}}^{(l)}}$;
7: END WHILE
Output: $\bar{\mathbf{E}}^M\triangleq\hat{\mathbf{E}}^*$.

Proposition 2. The sequence $\left\{\widehat{\mathbf{GEE}}\left(\hat{\mathbf{E}}^{(l)},\bar{\mathbf{x}}^M\right)\right\}_l$ increases monotonically, and converges to the optimal solution $\bar{\mathbf{E}}^M$ of

[4] Note that for a fixed $\lambda\geq0$, $\bar{F}\left(\lambda\right)$ is concave in $\mathbf{Q}$.
[5] Proofs are eliminated due to space limitation, but can be found in [22].
[6] **Algorithm 2** has a polynomial-time complexity [15].

problem **GEE-PA-Stage-1**—satisfying Karush-Kuhn-Tucker (KKT) conditions—in a finite number of iterations [14].

B. Stage 2

Let the subcarrier assignment of the users in the SBS-tier be denoted by $\bar{\mathbf{x}}^S = \mathcal{M}^S$, obtained by the **DA** algorithm. Here, the aim is to optimize the network transmit energy of all users (in the MBS- and SBS-tiers). For notational convenience, let $\bar{\mathbf{x}} \triangleq \left(\bar{\mathbf{x}}^M, \bar{\mathbf{x}}^S\right)$—which corresponds to $\overline{\mathcal{M}} = \overline{\mathcal{M}}^M \cup \overline{\mathcal{M}}^S$—be the fixed subcarrier assignment for all network users. Hence, in Stage 2, problem **J-SA-GEE-PA** reduces to problem

GEE-PA-Stage-2:

$$\max \quad \mathbf{GEE}\left(\mathbf{E}, \bar{\mathbf{x}}\right)$$

$$\text{s.t.} \sum_{BS_m \in \mathcal{B}} \sum_{U_i \in \mathcal{U}_m} \bar{x}_{m,i}^k E_{m,i}^k \leq E^{\max}, \forall SC_k \in \mathcal{SC}, \tag{26a}$$

$$R_{0,i}\left(\mathbf{E}, \bar{\mathbf{x}}^M\right) \geq \mathbb{R}_{\min}^M, \forall U_i \in \mathcal{U}_0, \tag{26b}$$

$$R_{m,i}\left(\mathbf{E}, \bar{\mathbf{x}}^S\right) \geq \mathbb{R}_{\min}^S, \forall U_i \in \mathcal{U}_m, \forall m \geq 1, \tag{26c}$$

$$E_{m,1}^k \geq \cdots \geq E_{m,i}^k \geq \cdots \geq E_{m,|\mathcal{U}_m|}^k, \quad \forall SC_k \in \mathcal{SC}, \forall BS_m \in \mathcal{B}, \tag{26d}$$

$$\sum_{U_i \in \mathcal{U}_m} \sum_{SC_k \in \mathcal{SC}} \bar{x}_{m,i}^k E_{m,i}^k + E_{C,m} \leq \mathcal{E}_m, \forall BS_m \in \mathcal{B}, \tag{26e}$$

$$0 \leq E_{m,i}^k \leq \bar{x}_{m,i}^k E^{\max}, \quad \forall SC_k \in \mathcal{SC}, \forall U_i \in \mathcal{U}_m, \forall BS_m \in \mathcal{B}. \tag{26f}$$

It can be verified that problem **GEE-PA-Stage-2** is not convex. By utilizing the inequality in (19), and the variable substitution $E_{m,i}^k = 2^{Q_{m,i}^k}$, problem **GEE-PA-Stage-2** can be solved via **Algorithm 3**, until convergence to the network transmit energy matrix $\bar{\mathbf{E}}$.

VI. Two-Sided Exchange-Stability via Swap Matching

In the devised subcarrier assignment and GEE-maximizing power allocation algorithms, the preference lists of the users and subcarriers are initially constructed using a feasible transmit energy matrix, without accounting for intra-cell/intra-cell interference over each subcarrier. In other words, the obtained subcarrier assignment may no longer be stable, even after solving problems **GEE-PA-Stage-1** and **GEE-PA-Stage-2**. Hence, it essential to ensure stability of all users and subcarriers after subcarrier assignment and power allocation via the notion of two-sided exchange-stability [23].

Definition 8 (Swap Matching). Given a matching $\overline{\mathcal{M}}$, then users U_i and U_j (for $i \neq j$) swap their *subcarrier* assignments, while keeping the other user and subcarrier assignments unchanged, yielding the matching

$$\overline{\overline{\mathcal{M}}} = \left\{\overline{\mathcal{M}} \setminus \left\{\left(U_i, \overline{\mathcal{M}}\left(U_i\right)\right), \left(U_j, \overline{\mathcal{M}}\left(U_j\right)\right)\right\}\right\} \cup \left\{\left(U_i, \overline{\mathcal{M}}\left(U_j\right)\right), \left(U_j, \overline{\mathcal{M}}\left(U_i\right)\right)\right\}, \tag{27}$$

which involves swapping the allocated transmit energy to users U_i and U_j due to the swapped subcarriers.

For notational convenience, let $\bar{\bar{x}}$ and $\bar{\bar{\mathbf{E}}}$ denote the updated subcarrier assignment due to the obtained matching $\overline{\overline{\mathcal{M}}}$.

Definition 9 (Swap-Blocking Pair). A pair of subcarriers (SC_k, SC_l), for $k \neq l$ form a swap-blocking pair for $\overline{\mathcal{M}}$ iff:

(1) $\forall U_i \in \mathcal{U}_m$, $\forall BS_m \in \mathcal{B}$, $R_{m,i}\left(\bar{\bar{\mathbf{E}}}, \bar{\bar{x}}\right) \geq R_{m,i}\left(\bar{\mathbf{E}}, \bar{x}\right)$, and $\exists U_i \in \mathcal{U}_m$, for any $BS_m \in \mathcal{B}$, $R_{m,i}\left(\bar{\bar{\mathbf{E}}}, \bar{\bar{x}}\right) > R_{m,i}\left(\bar{\mathbf{E}}, \bar{x}\right)$, and

(2) $\mathbf{GEE}\left(\bar{\bar{\mathbf{E}}}, \bar{\bar{\mathbf{x}}}\right) > \mathbf{GEE}\left(\bar{\mathbf{E}}, \bar{\mathbf{x}}\right)$,

while ensuring that no subcarrier or base-station exceeds its total energy constraint.

Definition 10 (Two-Sided Exchange-Stability). A matching $\overline{\overline{\mathcal{M}}}$ is said to be two-sided exchange-stable if it does not contain any swap-blocking pairs.

A. Algorithm Description

Initially, for a given matching solution obtained via **Algorithm 1** over the two stages (i.e. $\overline{\mathcal{M}}$), each user searches for another user, and checks if a pair of subcarriers forms a swap-blocking pair. If a swap-blocking pair is found, a swap-operation is performed, and the matching is updated to $\overline{\overline{\mathcal{M}}}$. This process repeats until convergence, as in Algorithm 4.

Algorithm 4: Swap Matching

Input: Matching $\overline{\mathcal{M}}$.
1: FOR each user pair $(U_i, U_j) \in \mathcal{U}$
2: IF a pair (SC_k, SC_l) forms a swap-blocking pair
3: Perform a swap operation, and update matching to $\overline{\overline{\mathcal{M}}}$;
4: END IF
5: END FOR
Output: Updated matching $\overline{\overline{\mathcal{M}}}$.

B. Properties

1) Convergence: **Algorithm 4** converges to a matching $\overline{\overline{\mathcal{M}}}$ in a finite number of iterations.

2) Complexity: **Algorithm 4** has a polynomial-time complexity of $\mathcal{O}\left(K^2\right)$ per user pair.

It is noteworthy that **Algorithm 4** does not ensure stability just yet, as it only eliminates swap-blocking pairs, and updates the user-subcarrier assignment. However, by repeatedly executing **Algorithm 4** and updating the power allocation, a two-sided exchange-stable matching is obtained, as will be detailed in the following section.

VII. A Two-Stage Solution Procedure

In the first stage, **Algorithm 1** is used to obtain the initial subcarrier assignment matching solution for the MBS users (i.e. $\overline{\mathcal{M}}^M$). Then, **Algorithm 3** is executed to obtain the GEE-maximizing power allocation solution $\bar{\mathbf{E}}^M$ for the MBS users. In the second stage, the subcarrier assignment $\overline{\mathcal{M}}^S$ for the SBS users is similarly obtained by executing **Algorithm 1**. After that, **Algorithm 3** is executed to obtain the network GEE-maximizing power allocation solution $\bar{\mathbf{E}}$. To enforce stability, **Algorithm 4** is repeatedly executed to eliminate subcarrier swap-blocking pairs, and update the user-subcarrier assignment matching to $\overline{\overline{\mathcal{M}}}$, while optimizing the network GEE via **Algorithm 3**. This process repeats until convergence to $\mathbf{x}^*$ and $\mathbf{E}^*$, as outlined in Algorithm 5.

Proposition 3. Algorithm 5 converges to a two-sided exchange-stable matching $\mathbf{x}^*$ in a finite number of iterations, and the solution $(\mathbf{E}^*, \mathbf{x}^*)$ satisfies KKT conditions.

Algorithm 5: Solution Procedure for Joint Subcarrier Assignment and Global Energy-Efficient Power Allocation (SP-J-SA-GEE-PA)

Stage 1:

Construct $\mathcal{P}_{U_i}$ and $\mathcal{P}_{SC_k}$, $\forall U_i \in \mathcal{U}_0$, and $\forall SC_k \in \mathcal{SC}$;
Determine $\bar{\mathbf{x}}^M = \overline{\mathcal{M}}^M$ via **Algorithm 1**;
Solve **GEE-PA-Stage-1** via **Algorithm 3** to get $\bar{\mathbf{E}}^M$;

Stage 2:

Construct $\mathcal{P}_{U_i}$ and $\mathcal{P}_{SC_k}$, $\forall U_i \in \mathcal{U}_m$ ($\forall m \geq 1$), and $\forall SC_k \in \mathcal{SC}$;
Determine $\bar{\mathbf{x}}^S = \overline{\mathcal{M}}^S$ via **Algorithm 1**;
Solve **GEE-PA-Stage-2** via **Algorithm 3** to get $\bar{\mathbf{E}}$;
WHILE (a swap-blocking pair exists)
  Execute **Algorithm 4** and update matching to $\overline{\overline{\mathcal{M}}}$;
  Set $\bar{\bar{\mathbf{x}}}^* = \overline{\overline{\mathcal{M}}}$;
  Solve **GEE-PA-Stage-2** via **Algorithm 3** to get $\bar{\bar{\mathbf{E}}}^*$;
END WHILE

Output: $\mathbf{x}^* \triangleq \bar{\bar{\mathbf{x}}}^*$ and $\mathbf{E}^* \triangleq \bar{\bar{\mathbf{E}}}^*$.

VIII. Simulation Results

The simulations assume a total of $M = 3$ base-stations (i.e. a MBS, and 2 SBSs), as illustrated in Fig. 1. In addition, there is a total of $N = 16$ users, where the MBS includes users $\mathcal{U}_0 = \{U_1, \ldots, U_8\}$ (i.e. 8 users), $\mathcal{U}_1 = \{U_9, \ldots, U_{12}\}$, and $\mathcal{U}_2 = \{U_{13}, \ldots, U_{16}\}$ (i.e. 4 users in each SBS). Moreover, there is a total of $K = 8$ subcarriers, which can be utilized by all users within all base-stations. The total transmit energy per subcarrier is set to $E^{\max} = 0.5$ J. The noise variance is set to $N_0 = 10^{-8}$ J, while the path-loss exponent is set to $\nu = 3$. The target minimum rate per MBS user is set to $\mathbb{R}^M_{\text{min}} = 3$ bits/s/Hz, and $\mathbb{R}^S_{\text{min}} = 1.5$ bits/s/Hz for SBS users. The maximum harvested energy per time-slot at each base-station is set to $\mathcal{E}^{\max}_m = 1, 0.5$, and 0.5 J, for $m = 0, 1, 2$. The battery capacity is set to $B^{\max}_m = 5, 2.5$, and 2.5 J, while the fixed transceiver energy consumption $E_{C,m} = 0.01, 0.005$, and 0.005 J, for $m = 0, 1, 2$. Moreover, $\xi^M_k = \xi^S_k = 2$, $\forall SC_k \in \mathcal{SC}$, and $\zeta^M_i = \zeta^S_i = 2$, $\forall U_i \in \mathcal{U}_m$, and $\forall BS_m \in \mathcal{B}$. The simulations are averaged over 10^3 random network instances, each of 10 time-slots (i.e. $\tau = 1, \ldots, 10$), with randomly generated channel coefficients that remain constant during each network instance, but vary from one network instance to another. In addition, the **SP-J-SA-GEE-PA** is compared to the **J-SA-GEE-PA** scheme[7]. Additionally, **OFDMA** is compared to the aforementioned schemes, where it should be noted that in **OFDMA**, $\xi^M_k = \xi^S_k = 1$, $\forall SC_k \in \mathcal{SC}$. That is, in **OFDMA**, a subcarrier can be assigned to at most one user in each tier, while each user can be assigned to at most two subcarriers.

In Fig. 2a, one can see that the **SP-J-SA-GEE-PA** scheme yields slightly higher network sum-rate than the **J-SA-GEE-PA** and **OFDMA** schemes, at the expense of slightly higher energy consumption than the **J-SA-GEE-PA** scheme, but significantly less than **OFDMA** (see Fig. 2b). Consequently, the **SP-J-SA-GEE-PA** scheme yields slightly less (but comparable) network GEE to the **J-SA-GEE-PA** scheme, and significantly outperforms **OFDMA**, as shown in Fig. 2c. Thus, it would be expected that the average residual energy at the base-stations to be slightly less under the **SP-J-SA-GEE-PA** scheme than the **J-SA-GEE-PA** scheme, but significantly higher than **OFDMA**.

[7] All optimization problems are solved via MIDACO [24].

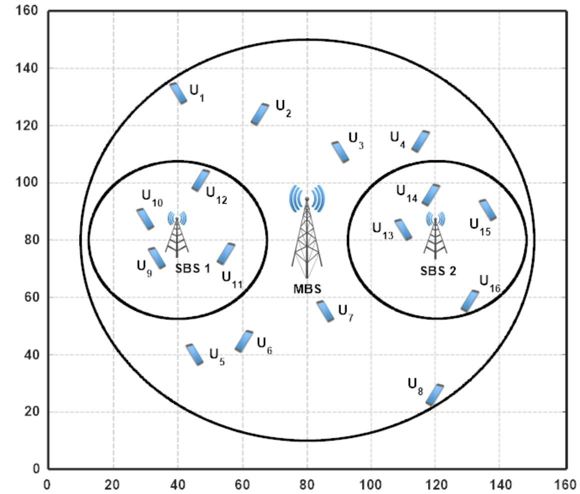

Fig. 1. Simulated Network Topology

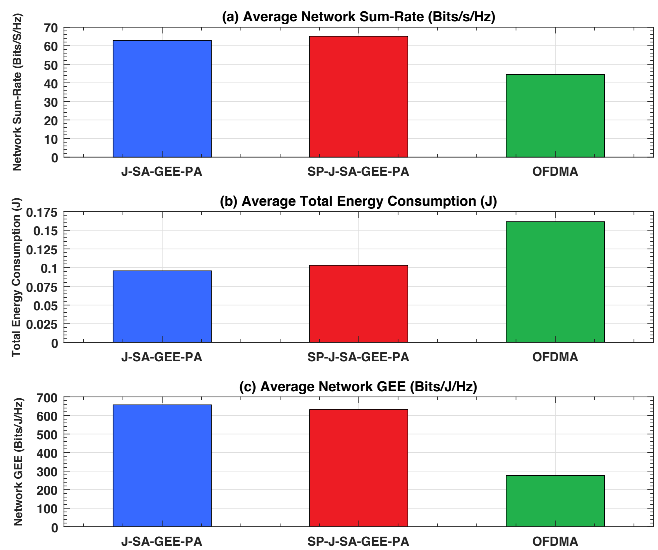

Fig. 2. (a) Average Network Sum-Rate (Bits/s/Hz), (b) Average Total Energy Consumption (J), and (c) Average Network GEE (Bits/J/Hz)

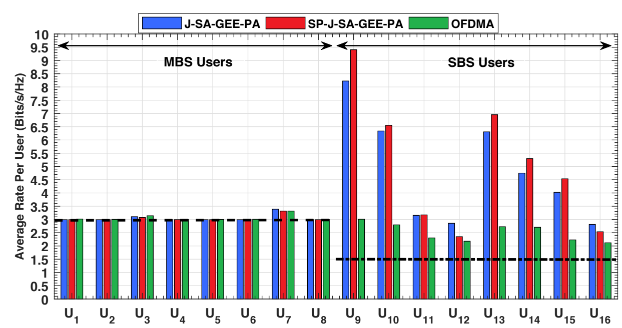

Fig. 3. Average Rate Per User (Bits/s/Hz)

Fig. 3 illustrates the average rate per user, where it is evident that all MBS users meet the target minimum rate of $\mathbb{R}^M_{\text{min}} = 3$ bits/s/Hz, while all SBS users significantly exceed their minimum rate requirement of $\mathbb{R}^S_{\text{min}} = 1.5$ bits/s/Hz. This is attributed to their locations being relatively closer to their respective base-stations than the MBS users.

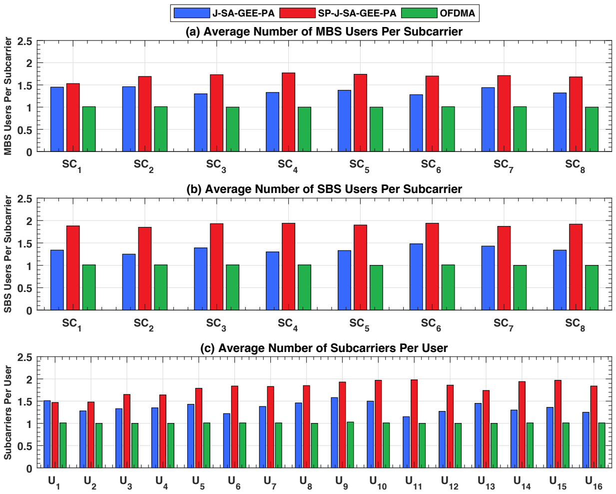

Fig. 4. Average Number of (a) MBS Users Per Subcarrier, (b) SBS Users Per Subcarrier, and (c) Subcarriers Per User

In Figs. 4a and 4b, it can be seen that the average number of MBS and SBS users per subcarrier is less than two, with the **SP-J-SA-GEE-PA** scheme assigning slightly higher average number of users to each subcarrier to ensure stability and maximize the GEE. Fig. 4c shows that the average number of subcarriers per user never exceeds two for the **SP-J-SA-GEE-PA** and **J-SA-GEE-PA** schemes, and one for **OFDMA**.

It has been verified via the simulations that **Algorithm 1** requires—on average—less than 5 (7) iterations in the MBS-tier (SBS-tier) for subcarrier assignment, while **Algorithm 3** less than 8 (10) iterations to find the optimal GEE-maximizing power allocation solutions in Stage 1 (Stage 2). Moreover, about 56% of the simulated instances involved between one and three swap operations by **Algorithm 4**. Hence, **Algorithm 5** can be executed efficiently to perform stable subcarrier assignment and GEE-maximizing power allocation.

IX. Conclusions

This paper has studied the problem of joint subcarrier assignment and global energy-efficient power allocation for energy-harvesting two-tier downlink NOMA HetNets, which has shown to be computationally-prohibitive. Thus, it has been split into two sub-problems. For the first sub-problem, the subcarrier assignment problem has been modeled as a many-to-many matching problem, while for the second sub-problem, the global energy-efficient power allocation has been solved optimally via a low-complexity algorithm. After that, a two-stage solution procedure has been proposed to efficiently solve the subcarrier assignment and power allocation sub-problems in the MBS and SBS tiers, while ensuring stability via swap matching. Simulation results have been presented to validate the proposed solution procedure, where it has been shown to yield comparable network energy-efficiency to the J-SA-GEE-PA scheme; however, with lower computational-complexity.

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