2019, 36th NATIONAL RADIO SCIENCE ONFERENCE
(NRSC 2019), April 16-18, 2019, Port Said, Egypt
Arab Academy for Science, Technology and Maritime Transport

OPTIMAL DOWNLINK RESOURCE ALLOCATION FOR JOINT TRANSMISSION COMP-ENABLED NOMA NETWORKS: A BENCHMARK IMPLEMENTATION

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ABSTRACT

Non-orthogonal multiple access (NOMA) is one of the most promising multiple access schemes, which is anticipated to improve the network spectral efficiency in 5G wireless cellular networks. Together with joint-transmission coordinated multi-point transmission (JT-CoMP), a JT-CoMP-NOMA network will also improve the data rates of cell-edge users, which are prone to severe inter-cell interference. In this paper we propose an implementation of the centralized resource allocation problem for JT-CoMP- NOMA in GAMS, where the aim is to perform joint sub-carrier assignment and power allocation in multi-cell downlink NOMA networks. The implementation serves as a benchmark for evaluating the network sum-rate performance of resource allocation algorithms for JT-CoMP-NOMA systems. Moreover, the provided implementation incorporates practical constraints, such as the maximum number of users multiplexed over each sub-carrier, SIC decoding order, user pairing, intra- and inter-cell interference, and minimum rate requirements. Simulation results are presented as a proof of concept.

Keywords: *Non-Orthogonal Multiple Access (NOMA), coordinated multi-point (CoMP), successive interference cancellation (SIC)*

I. INTRODUCTION

5G is the next generation wireless cellular network and is expected to support massive numbers of devices for both consumers and industry. It has been estimated that the mobile traffic is foreseen to increase in the order of 1,000 times over the next decade compared to what is experienced today [1]. In addition, the rapid development of the mobile Internet and the Internet of Things (IoT) calls for low-latency and high data-rate services. Therefore, 5G is expected to provide an end-to-end latency of 1ms, which is approximately one-fifth of the latency in 4G. Furthermore, 5G is expected to support a network throughput of 10 Gbps that is 10 times the peak data rate in 4G networks [2, 3].

In order to fulfil these requirements, there are several ways in which the spectral efficiency can be increased. One of the potential techniques that has gained tremendous interest in the research community and industry is the use of non-orthogonal multiple access (NOMA) for users multiplexing. In contrast to orthogonal frequency division multiple access (OFDMA) in which only one user equipment (UE) is served on each sub-carrier (SC), multiple UEs are multiplexed on the same SC simultaneously in NOMA. Multiplexing users over the same SC improves the spectral efficiency; however, it creates inter-user interference [4] that must be carefully managed. In this paper, we focus our discussion on the power-domain NOMA which exploits channel gain differences between the UEs and multiplexes them over the same SC through imbalanced/optimal power allocation [2, 3]. Thus, in a downlink NOMA system, a base station (BS) superposes all UEs' signals in the power domain according to their respective channel gains. To deal with the problem of inter-user interference, a multi-user detection (MUD) technique known as successive interference cancellation (SIC) [4] is implemented at the receiver side to detect the desired signal by successively cancelling the interfering signals of the weaker users.

Cellular networks are composed of several BSs, creating a multi-cell system. Therefore, in addition to intra-cell interference, cell-edge UEs are also vulnerable to inter-cell interference from neighboring cells, which further reduces their transmission rates. Coordinated multi-point (CoMP) transmission techniques are usually deployed in order to improve the data rates of cell-edge UEs. CoMP was introduced for downlink 4G cellular systems by the third-generation partnership project (3GPP) to mitigate inter-cell interference for cell-edge UEs [5], in which multiple neighboring BSs coordinate to schedule/transmit the signals to them. There are several CoMP schemes [6], where the most popular scheme is joint-transmission (JT)-CoMP. In this scheme, multiple BSs simultaneously transmit the same data to a cell-edge UE by using the same spectrum resources and hence, the reception performance for the UE is improved.

In this work, we propose an optimal implementation of the centralized resource allocation problem for JT-CoMP-NOMA networks with practical constraints in General Algebraic Modeling System (GAMS) [7, 8]. The proposed implementation serves as a benchmark for evaluating the network sum-rate performance of resource allocation algorithms for JT-CoMP- NOMA systems. Moreover, the provided implementation incorporates practical constraints, such as the maximum number of UEs multiplexed over each sub-carrier, SIC decoding order, user pairing, intra- and inter-cell interference, and minimum rate requirements.

The remainder of this paper is organized as follows. Section II presents the network model and problem formulation. Section III presents an overview of the benchmark implementation. MATLAB and GAMS implementations are discussed in Sections IV and V, respectively. Simulation results are given in Section VI, while Section VII draws the conclusions. The source codes are available at [10].

II. NETWORK MODEL

We consider a downlink NOMA network that supports JT-CoMP transmission. The multi-cell network consists of B BSs denoted by the set $\mathcal{B} = \{1, \cdots, B\}$ and U UEs denoted by the set $\mathcal{U} = \{1, \cdots, U\}$. The frequency spectrum allocated to a BS b is divided into K^b SCs and denoted by the set $\mathcal{K}^b = \{1, \cdots, K^b\}$. As mentioned earlier, in JT-CoMP systems all coordinated BSs transmit on the same SC, i.e., the frequency reuse is 1. Therefore, the set of SCs for all BSs in the network are equal, i.e., $\mathcal{K}^b = \mathcal{K}, \forall b \in \mathcal{B}$. The set of UEs associated with BS b is denoted by $\mathcal{U}^b$. The channel state information (CSI) is perfectly known to the BSs, and the channel coefficient from BS b to UE u on SC k is denoted by $h_{u,k}^b = g_{u,k}^b (d_u^b)^{-\alpha/2}$, where $g_{u,k}^b$ is an i.i.d. circular symmetric complex Gaussian random variable representing the Rayleigh fading, d_u^b is the propagation distance between BS b and UE u, and α is the path-loss exponent. The additive white Gaussian noise (AWGN) at UE u is denoted by n_u, which is zero-mean with variance σ_u^2. Therefore, the corresponding normalized channel gain of UE is defined as $\gamma_{u,k}^b = \frac{|h_{u,k}^b|^2}{\sigma_u^2}$.

In CoMP systems, UEs are categorized either as cell-edge users or cell-center users based on comparing their normalized average channel gains from BSs within their range to a threshold γ^{thr} [11]. A UE u with average channel gain satisfying the condition

$$\frac{1}{K^b} \sum_{k \in \mathcal{K}^b} \gamma_{u,k}^b \leq \gamma^{thr}, \qquad \forall b \in \mathcal{B}, \forall u \in \mathcal{U}^b, \tag{1}$$

in a given cell b is considered a cell-edge UE in that particular cell; otherwise, it is considered as a cell-center UE. The sets of cell-center and cell-edge UEs with respect to BS b are denoted by $\mathcal{T}^b$ and $\mathcal{E}^b$, respectively, given that $\mathcal{T}^b \cap \mathcal{E}^b = \emptyset$, $\mathcal{T} = \bigcup_{b \in \mathcal{B}} \mathcal{T}^b$ and $\mathcal{E} = \bigcup_{b \in \mathcal{B}} \mathcal{E}^b$. In addition, the set of CoMP users receiving joint transmissions from both BSs b and b', where $b \neq b'$, on SC k is denoted by $\mathcal{O}_k^{b,b'}$.

Recall that in NOMA systems a SC k in cell b can be allocated to multiple UEs which form a NOMA cluster denoted by $\mathcal{C}_k^b$. Moreover, each UE u can receive signals from the BS over multiple SCs [4, 12]. Over the same SC, UEs are multiplexed in the power domain by allocating different power levels to UEs. The power allocated to UE u in cell b over SC k is denoted by $p_{u,k}^b$. The allocated power to all UEs over the available spectrum in a given cell must satisfy the BS power budget constraint as follows

$$\sum_{\forall u \in \mathcal{U}} \sum_{k \in \mathcal{K}} p_{u,k}^b \leq \bar{P}^b, \qquad \forall b \in \mathcal{B}, \tag{2}$$

where $\bar{P}^b$ is the total available power budget of BS b over all SCs. A cell-center UE, $u \in \mathcal{T}^b$, receives a superposed signal of all UEs signals in the NOMA cluster over SC k, expressed as

$$y_{u,k}^b = h_{u,k}^b \sum_{\forall u' \in \mathcal{C}_k^b} \sqrt{p_{u',k}^b}\, x_{u',k}^b + n_{u,k}^b, \tag{3}$$

where $x_{u',k}^b$ is the signal of UE u' assigned to cluster $\mathcal{C}_k^b$, and $n_{u,k}^b$ is the received AWGN. Unlike cell-center UEs, a cell-edge UE, $u \in \mathcal{E}^b$, receives from its coordinated set of serving BSs, denoted by $\mathcal{S}_u$, a signal given by [13]

$$y_{u,k}^b = \sum_{\forall b \in \mathcal{S}_u} h_{u,k}^b \sum_{\forall u' \in \mathcal{C}_k^b} \sqrt{p_{u',k}^b}\, x_{u',k}^b + \phi_{u,k} + n_{u,k}^b, \tag{4}$$

where $\phi_{u,k}$ denotes the inter-cell interference received from BSs that are not in the coordinated set of u, as

$$\phi_{u,k} = \sum_{\forall b' \in \{\mathcal{B} \setminus \mathcal{S}_u\}} h_{u,k}^{b'} \sum_{\forall j \in \mathcal{C}_k^{b'}} \sqrt{p_{j,k}^{b'}}\, x_{j,k}^{b'}. \tag{5}$$

Note that for cell-center UEs $\phi_{u,k} = 0$ because they are either out of the coverage area of other cells or experience negligible inter-cell interference. It is also worthwhile to note that a cell-edge UE does not necessarily receive CoMP transmission. In this case, the cardinality of the set of coordinated BSs for cell-edge UE u is 1, i.e., $|\mathcal{S}_u| = 1$.

Since a SC $k \in \mathcal{K}$ is assigned to a given NOMA cluster $\mathcal{C}_k^b$, the signal of any UE $u \in \mathcal{C}_k^b$ causes interference to other UEs $u' \in \mathcal{C}_k^b$. To decode the intended message, each UE u performs successive interference cancellation (SIC) on the superposed signal. Let $\pi_k^b(u)$ be the order of UE u in a NOMA cluster $\mathcal{C}_k^b$ based on an increasing order of their channel gains as follows

$$\gamma_{u^1,k}^b < \gamma_{u^2,k}^b < \cdots < \gamma_{u^C,k}^b, \qquad \forall u \in \mathcal{C}_k^b \text{ and } C = |\mathcal{C}_k^b|. \tag{6}$$

A UE $u \in \mathcal{C}_k^b$ successively decodes the signals of other UEs $j \in \{\mathcal{C}_k^b | \pi_k^b(j) < \pi_k^b(u)\}$ with lower channel gains and cancels them before it decodes its desired signal. However, signals of UEs $j' \in \{\mathcal{C}_k^b | \pi_k^b(j') > \pi_k^b(u)\}$ with larger channel gains are treated as intra-cell interference [12]. Therefore, according to SIC principle [2, 14], UEs with lower channel gains are allocated higher power levels, whereas UEs with higher channel gains are allocated lower power levels. Therefore, the following condition must hold for all UEs in a cluster $\mathcal{C}_k^b$

$$p_{u,k}^b \geq p_{j,k}^b, \qquad \forall u, j \in \{\mathcal{C}_k^b | \pi_k^b(u) < \pi_k^b(j)\}, u \neq j. \tag{7}$$

Let $\eta_{u,k}^b$ be a binary decision variable, such that

$$\eta_{u,k}^b = \begin{cases} 1, & \text{if UE } u \text{ is assigned SC } k \text{ in cell } b, \\ 0, & \text{otherwise.} \end{cases} \tag{8}$$

The achievable data rate for any UE $u \in \mathcal{U}$ being served by the set of BSs, $\mathcal{S}_u$, over SC $k \in \mathcal{K}$ is given by [6,12,13]

$$R_{u,k} = log_2\left(1 + \frac{\sum_{b\in\mathcal{S}_u} \eta_{u,k}^b\, p_{u,k}^b \gamma_{u,k}^b}{1 + I_{u,k} + \varphi_{u,k}}\right), \tag{9}$$

where $I_{u,k}$ and $\varphi_{u,k}$ denote the intra-cell and inter-cell interference terms, respectively, and are defined as follows

$$I_{u,k} = \sum_{b\in\mathcal{S}_u} \gamma_{u,k}^b \sum_{i\in\{\mathcal{C}_k^b | \pi_k^b(i) > \pi_k^b(u)\}} p_{i,k}^b, \tag{10}$$

and

$$\varphi_{u,k} = \sum_{b'\in\{\mathcal{B}\ \mathcal{S}_u\}} \gamma_{u,k}^{b'} \sum_{j\in\mathcal{C}_k^{b'}} p_{j,k}^{b'}. \tag{11}$$

Note that for a cell-center UE $u \in \mathcal{T}^b$, the achievable data rate, intra-cell interference and inter-cell interference reduce to $R_{u,k} = \log_2\left(1 + \frac{\eta_{u,k}^b p_{u,k}^b \gamma_{u,k}^b}{1+I_{u,k}}\right)$, $I_{u,k} = \gamma_{u,k}^b \sum_{i\in\{\mathcal{C}_k^b|\pi_k^b(i)>\pi_k^b(u)\}} p_{i,k}^b$ and $\varphi_{u,k} = 0$, respectively, since $|\mathcal{S}_u| = 1$ and inter-cell interference is negligible. To guarantee QoS for all UEs on their assigned SCs, $R_{u,k}$ must be at least R_{min},

$$R_{u,k}^b \geq R_{min}. \tag{12}$$

In order to successfully perform SIC, the UEs scheduled in a NOMA cluster must have distinct channel gains [15]-[17], i.e., the channel gain difference between any two UEs $u, u' \in \mathcal{C}_k^b$, $u \neq u'$, must be greater than threshold ν, which can be written as

$$|\gamma_{u,k}^b - \gamma_{u',k}^b| > \nu, \qquad \forall b \in \mathcal{B}, \forall k \in \mathcal{K}, \forall u, u' \in \mathcal{C}_k^b, u \neq u'. \tag{13}$$

Considering the complexity of SIC, which is in the order of $O\left(|\mathcal{C}_k^b|^3\right)$ [12, 18], the number of UEs multiplexed over a SC is limited to κ. Furthermore, the number of SCs assigned to a given UE is limited to μ. Therefore, the following constraints must be satisfied

$$\sum_{u\in\mathcal{U}} \eta_{u,k}^b \leq \kappa, \qquad \forall k \in \mathcal{K}, \tag{14}$$

and

$$\sum_{k\in\mathcal{K}} \eta_{u,k}^b \leq \mu, \qquad \forall u \in \mathcal{U}. \tag{15}$$

Furthermore, considering the joint-transmission latency and overhead between coordinated BSs, the number of serving BSs for a cell-edge UE $u \in \mathcal{E}$ is limited to τ. On the other hand, the maximum number of serving BSs for a cell-center UE, $u \in \mathcal{T}$, is 1. Thus, the following conditions arise

$$|\mathcal{S}_u| \leq \tau, \qquad \forall u \in \mathcal{E}, \tag{16}$$

and

$$|\mathcal{S}_u| = 1, \qquad \forall u \in \mathcal{T}. \tag{17}$$

As UEs are scheduled on non-orthogonal spectrum resources, there could be more than one cell-edge UE receiving CoMP transmission on SC k from the same set of coordinated BSs $b, b' \in \mathcal{B}, b \neq b'$, i.e., $\left|\mathcal{O}_k^{b,b'}\right| > 1$. A CoMP UE $u \in \mathcal{O}_k^{b,b'}$ in NOMA clusters $\mathcal{C}_k^b$ and $\mathcal{C}_k^{b'}$ will receive its desired signal by applying SIC according to its decoding orders $\pi_k^b(u)$ and $\pi_k^{b'}(u)$ in each cluster, respectively. Nevertheless, according to [6], if a NOMA cluster has more than one CoMP UE, the following two conditions are necessary:

- All non-CoMP UEs in NOMA clusters $\mathcal{C}_k^b$ and $\mathcal{C}_k^{b'}$ should have higher decoding order than UEs receiving CoMP transmissions from BSs b and b'. In other words, non-CoMP UEs of both clusters should be able to decode the signals of all CoMP UEs in $\mathcal{O}_k^{b,b'}$ before decoding their desired signals.
- CoMP UEs forming NOMA clusters in multiple cells keep the same decoding order in all cells regardless of the order of their channel gains.

Accordingly, for a CoMP UE to successfully perform SIC on a SC that it is receiving simultaneous signals on, it should maintain the same decoding order in both cells regardless of its channel gains in these cells. Let $\pi_k^b(u)$ and

$\pi_k^{b'}(u)$ be the decoding order of CoMP UE $u \in \mathcal{O}_k^{b,b'}$ on SC $k \in \mathcal{K}^b, \mathcal{K}^{b'}$ receiving transmission from BSs $b, b' \in \mathcal{B}, b \neq b'$, then

$$\pi_k^b(u) = \pi_k^{b'}(u) \qquad \forall u \in \mathcal{O}_k^{b,b'}, \forall k \in \mathcal{K}, \forall b, b' \in \mathcal{B}, b \neq b'. \tag{18}$$

The overall network performance can be evaluated by the total sum-rate of all users in all cells over all SCs, as given by

$$R_{total} = \sum_{u \in \mathcal{U}} \sum_{k \in \mathcal{K}} R_{u,k}. \tag{19}$$

Based on the aforementioned model, we formulate the sum-of-rates (**SOR**) optimization problem, with the objective to maximize the sum of UEs' rates in the multi-cell system as follows

$$\boldsymbol{SOR}: \quad \max_{p_{u,k}^b, \eta_{u,k}^b} R_{total}$$

Subject to (2), (7), (8), (12), (13), (14), (15), (16), (17), (18)

The formulated problem is a mixed-integer nonlinear programming (MINLP) problem, which is non-convex and combinatorial in nature, due to the existence of the interference terms in the objective function and the binary decision variables for SC allocation [19]. In turn, this problem is NP-hard, and there is no computationally-efficient approach to solve this problem optimally [20].

III. A General Overview of the Implementation

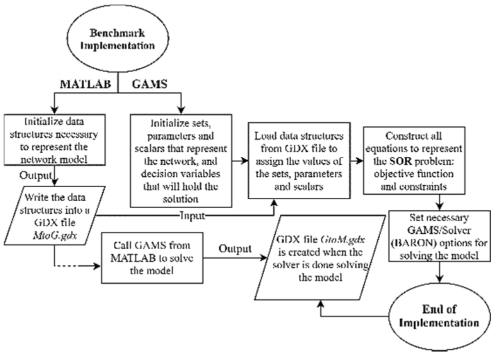

Fig. 1: Benchmark Implementation Flowchart.

In this section we present a general overview of the proposed GAMS implementation of the **SOR** problem. The implementation allows us to solve the problem under practical network constraints and settings. Therefore, our implementation serves as a benchmark for assessing the performance of resource allocation algorithms for JT-CoMP-NOMA networks developed for solving the **SOR** problem. Fig. 1 shows a flowchart for a high-level representation of our benchmark implementation. In GAMS, the mathematical equations are modeled based on static parameters that should be supplied before solving the problem. MATLAB will be used to initialize the static parameters and retrieve the results, whereas the problem will be modeled in GAMS. To import and export data efficiently from GAMS to MATLAB and vice versa, GAMS Data Exchange (GDX) files are used [21]. This is done using two important MATLAB routines—that become available after the installation of GAMS—namely, `rgdx` and `wgdx`. The former is used to read data from GDX files that contain the GAMS output into MATLAB, and the latter is used to write MATLAB structures into GDX files for loading network parameters into GAMS.

IV. MATLAB Implementation

The main functions of the MATLAB implementation are presented in this section.

A. Introducing GDX Data Structures

The input parameters sent to GAMS have to be written in a GDX file. One of the most important features of a GDX file is its Unique Element List (UEL). GDX files have only one global UEL, and every element in it is mapped to this UEL. Note that a user can enter a local UEL for each symbol and `wgdx` will create a global UEL; otherwise a default UEL ranging from 1 to n will be created for the elements [21]. For reading and writing symbols into a GDX file, the input arguments to the `rgdx` and `wgdx` routines are MATLAB structures composed of specific field names that are readable by the GAMS system. The reader is referred to [21] for more details on the format of the structure.

B. Building GDX Structures for the NOMA-CoMP Model

For GAMS to solve the model, we need to provide it with the network parameters. Given the formats of the input structures, MATLAB variables can be easily written into a GDX file using `wgdx` routine.

Our network model is mainly composed of BSs, UEs and SCs. Conventionally, such information is usually represented as sets [22]. In addition to the set of UEs $\mathcal{U}$, recall that in CoMP systems UEs are categorized into cell-center and cell-edge UEs, represented by the sets $\mathcal{T}$ and $\mathcal{E}$, respectively. Therefore, two set structures are required to represent $\mathcal{T}$ and $\mathcal{E}$.

On the other hand, for representing the SCs in the network, we are going to need two different set structures. Recall that the frequency reuse is 1 and thus, all BSs reuse the same SCs, i.e., $\mathcal{K}^b = \mathcal{K}, \forall b \in \mathcal{B}$\$. Therefore, the set of SCs $\mathcal{K}$ will represent the SC identification (ID) in the whole network, regardless to which BS it belongs to, `scidStruct.uels = {'scid1', 'scid2', ..., 'scidK'}`. To distinguish between the SCs that belong to BSs b and b', $b \neq b'$, the set structure `scStruct` is constructed using, `scStruct.uels = {'sc1', 'sc2', ..., 'scQ'}`. The `uels` of `scStruct` is a cell of size 1xQ, where $Q = K \cdot B$. This means, the first K elements in `scStruct.uels` represent the SCs that belong to the first BS, the second K elements represent the SCs that belong to the second BS, and so on.

Let the mapping of SCs to BSs be expressed in a 2-dimensional (2D) binary adjacency matrix `BsSc` of size BxQ. The element `BsSc(b,sc)` = 1 if SC sc belongs to BS b, and 0 otherwise. Here, the `bsScStruct.val` is assigned to the SC mapping matrix `BsSc`. To copy the whole matrix without supressing the zeros we need to set `bsScStruct.form = 'full'`. The `uels` field is set to the that of `bsStruct` and `scStruct` because the rows of matrix `BsSc` represent the BSs and the columns represent the SCs, `bsScStruct.uels = {bsStruct.uels, scStruct.uels}`. Notice that the *type* of `bsScStruct` is set to `'set'`. Unlike the previous set structures that we have created, `bsScStruct` is a multi-dimensional set with a many-to-many mapping [7]. It is a mapping between the elements of `bsStruct` and `scStruct`, which is provided by `BsSc` matrix.

Likewise, we need to create a mapping between the SC IDs in `scidStruct` to each SC in every BS in the network, i.e., `scStruct`. Let `ScidSc` be a 2D binary matrix of size K x Q which represents mappings. Subsequently, `scid_scStruct` is constructed the same way as `bsScStruct`.

The set/parameter structures described above initialize the main structures that describe the network model. The remaining necessary parameters that need to be created in order to represent all network variables described in Section II can be found at [10]. For writing/reading the GDX files and calling GAMS from MATLAB the reader is referred to [22].

V. GAMS IMPLEMENTATION

This section describes the details of the implementation and modeling of the **SOR** problem in GAMS, setting the solver options, reading the network parameters from the `MtoG.gdx` file and calling the solver.

A. Declaration of Sets and Aliases

In this sub-section we present the declaration of sets that represent the network elements. Particularly, the set of BSs $\mathcal{B}$, UEs $\mathcal{U}$ and SC ID $\mathcal{K}$, in addition to the sets described in Sub-section IV-B. The syntax used in GAMS for declaring sets are `Sets ue, bs, scid, sc, cue(ue), eue(ue), scid_sc(scid,sc), bs_sc(bs,sc), c(ue,bs)`. Note that some sets such as the set of cell-center UEs `cue`, for instance, is declared as a subset of the set of UEs `ue`. In other words, the declaration `cue(ue)` means that each member of the set `cue` must also be a member of the set `ue`.

In our **SOR** problem, the desired output of the solver is an optimal resource allocation that maximizes the network sum-rate, which is simply the summation of the data rates of all UEs in the network. Due to the interference terms in equation (9), the rate of each UE is affected by the other UEs in the network assigned to the same SC. To accommodate for this effect, it is important to redefine the set of UEs `ue` using the `Alias` command to provide a secondary domain for the solver [7, 22]. The command `Alias(ue,uep)` creates another domain for the set `ue` to encompass each UE and all its possible interferes in the set `uep`.

B. Declaration of Parameters and Scalars

In GAMS, parameters and scalars are static variables that do not change while the model is being solved. In fact, the solver uses these parameters in order to find the solution. Parameters are multi-dimensional variables, and their domains are specified in brackets following the parameter's name. On the other hand, a scalar is a special case of a parameter of dimensionality of zero [7]. This means it is not declared over a specific domain, and thus there is exactly one number associated with the parameter.

To declare parameters, the `Parameters` command is used to represent channel gains (`gamma(ue,sc)`), UE quotas for maximum number of assigned SCs (`quota_ue(ue)`), SC quotas for the maximum number of multiplexed UEs (`quota_sc(sc)`), BSs transmit power (`bs_tx_power(bs)`), and UE pairing threshold (`uepairing_thr(sc)`). Further, the maximum number of serving BSs for cell-center (`cue_serving_bs`) and cell-edge UEs (`cue_serving_bs`), and the minimum rate requirement threshold (`ratemin`) are declared scalars using the `Scalars` keyword.

The values of the sets, parameters and scalars are imported from the GDX file that was created in Section IV by using the `wrgdx` routine. The GAMS commands required for loading the GDX file can be found in [22].

C. Declaration of Variables

Variables in GAMS are decision variables which hold the solution and their values remain unknown until the model is solved. Like the parameters, variables can be declared dimensionless or multi-dimensional, with their domains specified over sets. The following variables are declared for the **SOR** problem:

- A 0-dimension objective variable representing the total network sum-rate, i.e., the objective value of the **SOR** problem: `Variable sumrate`.
- A 2D binary variable representing the SC assignment for UEs, $\eta_{u,k}^{b}$: `Binary Variable eta(ue,sc)`. Similarly, two more binary variables are needed, namely `bsserving(ue,bs)` and `ueserved(ue,scid)`. The former indicates from which BS each UE is receiving transmission from, while the latter represents on which SC ID each user is multiplexed on regardless of the BS.
- A 2D positive variable for the power allocation for UEs, $p_{u,k}^{b}$: `Positive Variable p(ue,sc)`.

D. Declaration of Equations

A GAMS equation consists of symbolic algebraic relationships that will be used to produce the objective function and constraints. The syntax `sum(domain, variable)` is used to iterate and sum over all involved variable domains. In addition, the syntax term `$logical_condition` is used to allow for exceptions when calculating or specifying a domain [7]. In the following we will model the some of the **SOR** problem equations in GAMS.

The power allocation for UEs in a cluster must follow the SIC principle for decoding as in (7). Moreover, recall that the decoding order of CoMP UEs must be kept the same in all clusters formed at all serving BSs, regardless of their channel gains in each BS as in (18). It is clear that the power allocation and the decoding order of UEs are related, and thus are modeled together in the same equation in GAMS. However, note that due to the several conditions and assignment possibilities, this equation is divided into sub-equations to model all possibilities. Following are some of the SIC power constraints. All remaining can be modeled in a similar manner following the same logic.

1) ***Between cell-center UEs:***

```
p_sic_constraint_1(ue,uep,bs,sc)$(cue(ue)$YES and cue(uep)$YES and gamma(ue,sc) > gamma(uep,sc)
and ord(ue) <> ord(uep) and bs_sc(bs,sc)$YES and c(ue,bs)$YES and c(uep,bs)$YES) ..
bsserving(ue,bs)*bsserving(uep,bs)*eta(ue,sc)*eta(uep,sc)*p(ue,sc) =L= p(uep,sc);
```

2) ***Between cell-center and cell-edge UEs:***

```
p_sic_constraint_2(ue,uep,bs,sc)$(cue(ue)$YES and eue(uep)$YES and ord(ue) <> ord(uep) and
gamma(ue,sc) > gamma(uep,sc) and bs_sc(bs,sc)$YES and c(ue,bs)$YES and c(uep,bs)$YES) ..
bsserving(ue,bs)*bsserving(uep,bs)*eta(ue,sc)*eta(uep,sc)*p(ue,sc) =L= p(uep,sc);
```

3) ***Between cell-edge UEs having ONLY two BSs in their range:*** if both UEs are CoMP, if both UEs are non-CoMP, and if one of the UEs is CoMP and the other is non-CoMP.

As stated in Section II, all non-CoMP UEs in a cluster should have higher decoding order than all UEs—in that cluster—receiving CoMP. Like the SIC power constraint, this constraint is modeled using several sub-equations for the same reasoning. Here we will only present the modeling of the constraint between cell-center and cell-edge UEs. The equation declaration creates constraints for cell-center UEs `ue` and cell-edge UEs `uep`, where `gamma(ue,sc) < gamma(uep,sc)`. In addition, `ue` and `uep` belong to the same cell `bs`, where `uep` also has `bsp` in its range. Since `gamma(ue,sc) < gamma(uep,sc)`, then if `uep` is assigned CoMP, `ue` has the option to either be served by `bs` but on another SC, or not to be served at all by `bs`. This part of the equation is written as,

```
{ueserved(uep,scid)*bsserving(uep,bs)*bsserving(uep,bsp)*eta(uep,sc)*eta{uep,scp)}*{[1 - ueserved(ue,scid)]*
bsserving(ue,bs)*[1 - eta(ue,sc)] + [1 - ueserved(ue,scid)]*[1 - bsserving(ue,bs)]*[1 - eta(ue,sc)]]}
```

On the other hand, if `uep` is: 1) assigned non-CoMP from either `bs` or `bsp`, 2) not served by any of its BSs, 3) assigned CoMP on a SC having a SC ID other than `scid`, or 4) assigned non-CoMP on either `bs` or `bsp` but on a SC having a SC ID other than `scid`. Then, `uep` can have either of the following assignments: 1) served by `bs` over `sc`, 2) served by `bs` over a different SC, or 3) not served by `bs`. Subsequently, this part of the equation is written as follows,

```
{ueserved(uep,scid)*bsserving(uep,bs)*[1 - bsserving(uep,bsp)]*eta)uep,sc)*[1 - eta(uep,scp)] +
ueserved(uep,scid)*[1 - bsserving(uep,bs)]*bsserving(uep,bsp)*[1 - eta(uep,sc)]*eta(uep,scp) + [1 -
ueserved(uep,scid)]*[1 - bsserving(uep,bs)]*[1 - bsserving(uep,bsp)]*[1 - eta(uep,sc)]* [1 - eta(uep,scp)] +
[1 - ueserved(uep,scid)]*bsserving(uep,bs)*bsserving(uep,bsp)*[1 - eta(uep,sc)]*[1 - eta(uep,scp)] + [1 -
ueserved(uep,scid)]*bsserving(uep,bs)*[1 - bsserving(uep,bsp)]*[1 - eta(uep,sc)]*[1 - eta(uep,scp)] + [1 -
ueserved(uep,scid)]*[1 - bsserving(uep,bs)]*bsserving(uep,bsp)*[1 - eta(uep,sc)]*[1 -eta(uep,scp)]}*
{ueserved(ue,scid)*bsserving(ue,bs)*eta(ue,sc) + [1 - ueserved(ue,scid)]*bsserving(ue,bs)*[1 - eta(ue,sc)] +
[1 - ueserved(ue,scid)]*[1 - bsserving(ue,bs)]*[1 - eta(ue,sc)]}
```

To model the minimum rate constraint (12), the GAMS equation is modeled to take into account the different possibilities of cell-center, cell-edge CoMP and cell-edge non-CoMP UE assignments, together with the different interference terms for each assignment. Due to the large number of code-lines required to model the constraint, the logic of writing the equation will be described by presenting few parts of the equation only. The following GAMS equation creates a minimum rate constraint for each UE over each SC.

```
minrate_constraint(ue,scid) .. log(1 + sum[(bs,sc), (ueserved(ue,scid)*bsserving(ue,bs)*eta(ue,sc)*p(ue,sc)*
```

```
gamma(ue,sc))$(scid_sc(scid,sc)$YES and bs_sc(bs,sc)$YES and c(ue,bs)$YES)]/
[1 + intra_cell + inter_cell])/log(2) =G= ueserved(ue,scid)*ratemin;
```

The `intra_cell` and `inter_cell` terms are just symbolic names for representation purposes in this paper. The `inter_cell` term is easily modeled as in equation (11) as follows,

```
{sum[(bs,sc), (ueserved(ue,scid)*[1 - bsserving(ue,bs)]*[1 - eta(ue,sc)]*gamma(ue,sc))$(scid_sc(scid,sc)$YES
and bs_sc(bs,sc)$YES and c(ue,bs)$YES)*sum[uep, (ueserved(uep,scid)*bsserving(uep,bs)*eta(uep,sc)*p(uep,sc)$
(c(uep,bs)$YES and ord(ue) <> ord(uep))]]}${eue(ue)$YES}
```

Notice that the inter-cell interference is only computed for cell-edge UEs, and thus the whole block representing the `inter_cell` term is enclosed in a pair of braces and followed by the $-operator `${eue(ue)$YES}`. On the other hand, the modeling of the `intra_cell` term requires decomposing it into several parts to account for cell-center, non-CoMP cell-edge, and CoMP cell-edge UE assignments. Therefore, the `intra_cell` term can be written as a summation of multiple terms representing all different parts, such as `intra_cell = term_1 + term_2 + ... + term_N`.

1) ***Intra-cell interference for cell-center UEs.***
2) ***Intra-cell interference for cell-edge UEs NOT receiving CoMP:*** interference from cell-center UEs and interference from cell-center UEs.
3) ***Intra-cell interference for cell-edge UEs receiving CoMP transmissions:*** interference from other CoMP cell-edge UEs and interference from non-CoMP UEs.

Finally, the objective function is written exactly the same way as the minimum rate equation, except that there is a summation over the domains `ue` and `scid` to calculate the total network sum-rate.

E. GAMS Options

GAMS provide the `option` statement to set global system parameters; for instance, to control the output details and set solver specific options [7]. GAMS can solve the model using a variety of external solvers. To solve the MINLP **SOR** problem, the following option specifies BARON to be used as a solver: `Option MINLP = BARON`. To guarantee the optimality of the solution, the `optcr` and `optca` options are set to 0.0. The former is the relative optimality criterion, or simply the relative gap, whereas the latter is the absolute optimality criterion, or simply the absolute gap. In addition, to ensure that the solver runs enough time to arrive at a solution, we set the wall-clock time limit option to approximately 2 months: `Option RESLIM = 5500000` (in seconds).

F. Calling the Optimization Problem Solver

GAMS itself does not solve optimization problems; however, it prepares the model and passes it to an external solver. As mentioned earlier, BARON is used to solve the **SOR** problem due to its MINLP nature. The following command creates the `nomacompnet` model, `Model nomacompnet /all/`. The commad `all` passes all constraints/equations to the solver to be solved. After creating the model, the `Solve` statement is used to call the solver by specifying the model name, the type of the problem and the variable to be optimized: `Solve nomacompnet using MINLP maximizing sumrate`.

VI. SIMULATION RESULTS

In this section we present the simulation results for the proposed benchmark implementation. We test the implementation by setting up a small network[1] as shown in Fig. 2 and compare the results with that of optimal OFDMA scheme. The network is composed of 3 circular cells and a total of 9 UEs distributed among the cells. The BSs are positioned in the center of the cells and each cell has 5 UEs within its coverage area. The frequency spectrum allocated to each BS is divided into 3 SCs, therefore, a total of 9 SCs in the whole network. We set the maximum number of SCs assigned to each UE to $\mu = 2$. Furthermore, due to SIC complexity we allow a maximum of 2 UEs to be multiplexed on each SC, i.e., $\kappa = 2$. We set the BSs' peak power, $\bar{P}^b$ to 40 dBm $\forall b \in \mathcal{B}$, the noise power to -30 dBm/Hz, the path-loss exponent $\alpha = 3$, and the minimum rate $R_{min} = 0.5$ bps/Hz. In addition, we set the maximum number of serving BSs for cell-edge UEs, to 2 i.e., $\tau = 2$ and the channel gain difference for user pairing ν is set to the standard deviation of the UEs channel gains.

[1] A small network topology was chosen due to the NP-hardness of the formulated MINLP SOR problem and the considerable amount to time needed to arrive at the optimal solution.

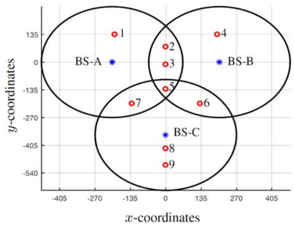

Fig. 2: Network topology. BS locations are labelled by asterisk at the center, while UEs locations are labelled by circles.

Network Instance	*BS*	SC_1	SC_2	SC_3
1	BS-A	(3)	(1,5)	(1)
	BS-B		(4,5)	(2,4)
	BS-C	(9)	(6,9)	(7,8)
2	BS-A	(1,7)	(1)	(2)
	BS-B	(3,4)	(4,5)	
	BS-C	(8)	(9)	(6,8)
3	BS-A	(1)	(7)	(1,5)
	BS-B	(3)	(2,4)	(4,5)
	BS-3	(6,8)	(8)	(9)

Fig. 3: User groups in each BS over each SC for different network instances.

The GAMS model was tested for three network instances. Fig. 3 shows the user grouping in each BS over each SC, for the JT-CoMP-NOMA scheme[2]. The numbers between the brackets represent the UEs multiplexed on SC k (denoted by SC_k for $k = 1,2,3.$). It is observed that UE5 was assigned CoMP transmission in instances 1 and 3. In instance 1, UE5 is receiving its signal from coordinated BSs A and B over SC_2. UE5 is in NOMA clusters $\mathcal{C}_2^A$ and $\mathcal{C}_2^B$ and is paired with UE1 in the first cluster and with UE4 in the second, where both UE1 and UE4 are cell-center UEs. Therefore, their channel gains are greater than that of UE5, which is one of the conditions that should be satisfied for CoMP users. Note that no other UE is receiving CoMP transmission in instance 1; however, SC_1 in BS-B is kept idle to reduce inter-cell interference on cell-edge UE3 which is being served by BS-A on SC_1. All other cell-edge UEs are served by achieving a data rate of 0.5 bps/Hz (see Fig. 4(b)). Similarly, in instance 3, UE5 is assigned CoMP transmission from BSs A and B over SC_3. However, in instance 3, no SC is kept idle since all other cell-edge UEs are assigned enough power from their serving BSs to satisfy the minimum rate requirement. On the other hand, in instance 2 there was no need to enable CoMP transmission for any cell-edge UE, since every UE is allocated enough power to satisfy the minimum rate requirement.

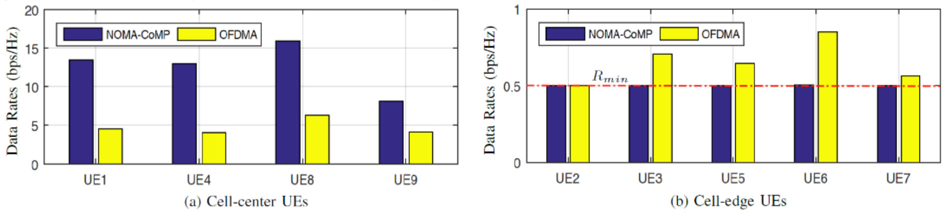

Fig. 4: Average users data rates over three network instances.

Fig. 4(a) and 4(b), respectively, show the average data rates for cell-center and cell-edge UEs taken over the three network instances for both NOMA-CoMP and OFDMA schemes. For NOMA-CoMP, notice that cell-center UEs are able to achieve high data rates due to their high channel gains. Moreover, NOMA shows a significant improvement for cell-center UEs over OFDMA. This is due to the fact that NOMA allows multiple UEs to be multiplexed on the same SC and each UE to be assigned multiple SC, and hence increasing the network's spectral efficiency. On the other hand, due to the bad channel conditions of cell-edge UEs and the intra-cell interference introduced by sharing resources, they are only able to achieve the minimum rate requirement in NOMA. However, since in OFDMA they experience an interference-free transmission, they are able to achieve slightly higher rates. This shows that there is a trade-off when using NOMA, where the spectral efficiency is improved on the disadvantage of reducing the rates of cell-edge UEs. Nevertheless, by introducing CoMP, cell-edge UEs are able to combat the interference by receiving joint transmission form multiple BSs. The NOMA-CoMP network sum-rate for instances 1, 2 and 3 is 50.349, 50.369 and 58.167 bps/Hz, respectively. Whereas, on the other hand, the OFDMA network sum-rate for the three instances is 20.646, 21.753 and 24.966 bps/Hz, respectively.

[2] Note that only the user-subcarrier assignment for the NOMA scheme is presented (Fig. 3) to focus on the main scope and results of the implementation in this paper.

VII. Conclusions

In this paper we presented a detailed benchmark implementation of the centralized resource allocation in JT-CoMP-NOMA networks. We modeled the sum-of-rates problem as a MINLP problem. Practical constraints, namely SIC complexity, decoding order, and user pairing, intra- and inter-cell interference, and minimum rate requirements were considered. Therefore, our implementation can be used as a benchmark for evaluating the performance of resource allocation algorithms for JT-CoMP-NOMA systems. The model is implemented in GAMS and solved by BARON, while model parameters were set in MATLAB. The implementation was tested on a small network set-up as proof of concept.

VIII. Acknowledgment

Authors acknowledge the support of the Kuwait Foundation of the Advancement of Sciences (KFAS).

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