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Abstract—In this paper, we study resource management and allocation for Energy Harvesting Cognitive Radio Sensor Networks (EHCRSNs). In these networks, energy harvesting supplies the network with a continual source of energy to facilitate self-sustainability of the power-limited sensors. Furthermore, cognitive radio enables access to the underutilized licensed spectrum to mitigate the spectrum-scarcity problem in the unlicensed band. We develop an aggregate network utility optimization framework for the design of an online energy management, spectrum management and resource allocation algorithm based on Lyapunov optimization. The framework captures three stochastic processes: energy harvesting dynamics, inaccuracy of channel occupancy information, and channel fading. However, a priori knowledge of any of these processes statistics is not required. Based on the framework, we propose an online algorithm to achieve two major goals: first, balancing sensors’ energy consumption and energy harvesting while stabilizing their data and energy queues; second, optimizing the utilization of the licensed spectrum while maintaining a tolerable collision rate between the licensed subscriber and unlicensed sensors. Performance analysis shows that the proposed algorithm achieves a close-to-optimal aggregate network utility while guaranteeing bounded data and energy queue occupancy. Extensive simulations are conducted to verify the effectiveness of the proposed algorithm and the impact of various network parameters on its performance.

Index Terms—Wireless Sensor Network, cognitive radio, energy harvesting, energy management, channel allocation, Lyapunov optimization

I. INTRODUCTION

Energy Harvesting Sensor Networks (EHSNs) are promising for long-term data collection over a wide range of applications [1], and become a fundamental enabling technology for the coming era of Big Data [2] and the Internet of Things (IoTs) [3], [4]. By exploiting the EH technology, sensors can harvest energy from the renewable energy sources in the area of interest, such as solar, illumination and vibration [5]. Therefore, it facilitates the self-sustainability of the power-constrained sensors and effectively extends the network lifetime.

EHSNs typically operate on the unlicensed Industrial, Scientific, and Medical (ISM) band for data transmission. However, the ISM band has become increasingly crowded due to the massive growth of wireless devices operating in this band. This massive growth has introduced the spectrum-scarcity problem which significantly degrades the performance of EHSNs. In addition, a large portion of the licensed spectrum remains underutilized, e.g., the spatial and temporal variations in the licensed spectrum utilization range from 15% to 85%, according to a report by Federal Communications Commission [6]. The integration of Cognitive Radio (CR) technology into EHSNs mitigates these licensed spectrum-underutilization and unlicensed spectrum-scarcity problems. It facilitates the transmission of sensed data over the underutilized licensed channels without disrupting the primary network operation. Such networks are referred to as Energy Harvesting Cognitive Radio Sensor Networks (EHCRSNs) in which sensors are secondary users (SUs) and primary network subscribers are primary users (PUs) [7]. The typical applications of EHCRSNs include the data collection indoors, where the sensors overlap with WiFi networks [8], the body sensor networks in pervasive health monitoring, and real-time monitoring in smart city [9]–[11].

Although EHCRSNs are spectrum and energy-efficient, they face several new challenges compared with the traditional sensor networks [12]. First, the energy harvesting process is stochastic and dynamic, which makes balancing energy consumption and energy replenishment challenging. Depleting a sensor’s battery at a rate slower or faster than the replenishment rate leads to either energy underutilization or sensor failure, respectively [13]. Second, the spectrum utilization by sensors in EHCRSNs has to adapt to the dynamic activity of PUs over the licensed spectrum [14]. For example, the spectrum occupation of cellular users is in the range of seconds or minutes [15]. When sensors transmit over the channels licensed to cellular users, the sensors may have to frequently disrupt their transmission and vacate the channels to avoid collisions with cellular users. Under these highly stochastic and dynamic conditions, managing and allocating resource for EHCRSNs becomes challenging.

To address the above challenges, we develop an aggregate utility optimization framework to facilitate the design
of an online algorithm that couples energy management with spectrum access management as well as sensing and transmission rate control for a single-hop EHCRSN. The considered EHCRSN consists of a sink and a number of sensors equipped with EH modules and CR transceivers. The sensors harvest energy to sense data and transmit it to the sink over the unoccupied licensed spectrum. The developed framework is Lyapunov optimization-based, and captures the dynamic and stochastic system of EHCRSN resources. Based on the framework, an online algorithm is designed to achieve a close-to-optimal time-average aggregate network utility, which captures the data sensing efficiency of the network [16], while ensuring protection of PUs and a deterministic bound on the battery capacity of sensors. Summarily, the main contributions of this work are as follows:

1) We propose a stochastic formulation of the network utility optimization problem for the EHCRSN subject to the stability of sensors’ data queues and PUs’ protection. The proposed formulation accounts for the multiple dynamic and stochastic processes, including the energy consumption of data sensing and transmission, energy harvesting, PU activity on each channel and collisions with sensors, and channel fading.

2) We develop a framework to decompose the problem into three deterministic subproblems: battery management, sampling (i.e., sensing) rate control, and resource (i.e., channel and data rate) allocation on the basis of Lyapunov optimization. Under the developed framework, we propose an online and low-complexity algorithm which makes decisions at the beginning of each time slot and does not require any priori knowledge of the stochastic processes. Furthermore, we apply an unbalanced matching method to assign channels to sensors while considering the limited number of CR transceivers mounted on the sink.

3) We analyze the performance of the proposed algorithm in terms of PU protection and the stability of the sensors data queues. Furthermore, we compute the required battery capacity to support the operation of the proposed algorithm, which depends on the energy consumption of data sensing and transmission. The analysis shows that a higher network utility can be achieved at the cost of a larger data buffer and battery capacity.

The remainder of this paper is organized as follows. Related works are reviewed in Section II. The network model and problem formulation are presented in Section III. The proposed framework is presented in Section IV. Section V analyzes the stability and optimality of the proposed solution. Simulation results are provided to evaluate the performance of the proposed algorithm in Section VI. Section VII concludes this paper and outlines future work.

II. RELATED WORKS

Utility-optimal energy management policy design for EHWSNs has been widely addressed in the literature [5], [17]–[20]. In [19], Liu et al. design two algorithms to optimize the network utility by exploiting convexity of the network flow problem. The first algorithm computes the data sampling rate and routing based on dual decomposition. To deal with the fluctuations in the EH process, the other algorithm maintains the battery at a target level. In [5], Zhang et al. propose a distributed algorithm to schedule data sensing and perform routing for EHWSNs with limited battery capacity. Furthermore, the proposed algorithm mitigates the estimation error of the EH process by adaptively scheduling the data sensing and routing in each time slot. The authors of [20] present two algorithms for balanced energy allocation of sensors, and optimal data sensing and data transmission. In [5], [19] and [20], the authors assume a priori perfect knowledge of the harvesting process statistics. This may not be practical due to the stochastic nature of EH processes. Huang et al. design an online scheduling algorithm which jointly considers the data routing, admission control and energy management. The algorithm does not require priori knowledge of the EH process and achieves close-to-optimal utility for EHWSNs [18]. Based on the algorithm in [18], Xu et al. investigate the utility-optimal data sensing and transmission in EHWSNs with heterogeneous energy sources, i.e., power grids and harvested energy [17]. Xu et al. also study the trade-off between achieved network utility and cost on energy from power grid.

Other works exploit the spectrum utilization and performance improvement that CR technologies bring to WSNs and focus on channel allocation for CRSNs [21]–[24]. In [22], Ozger et al. propose an event-driven clustering protocol for event-to-sink communication coordination in CRSNs. The proposed protocol considers the availability of licensed spectrum in forming cluster, and minimizes the energy consumption for event detection. In [23], Li et al. investigate the cooperative spectrum sensing schedule for a CRSN, in which sensors decide whether to join spectrum sensing for energy conservation. An evolutionary game is formulated to facilitate the decision of sensors according to their utility history. The authors of [22] and [23], however, do not account for possible collisions between the PUs and SUs; they assume perfect knowledge of the spectrum occupancy. As a result, if spectrum sensing false alarms and detection errors are considered, these approaches cannot be adopted. Unlike in [22] and [23], the authors of [21] and [24] consider the imperfection of channel availability information and design channel allocation algorithms that guarantee the protection of PUs’ transmissions against collision. In [21], Urgaonkar et al. develop an opportunistic channel accessing policy for cognitive radio networks to maximize the network throughput by taking the maximum collision constraint in to account. In [24], Qin et al. optimize the delay and throughput of multi-hop secondary networks in which the secondary users are mounted with multiple CR transceivers.

The above-mentioned works either assume availability of spectrum and neglect the spectrum-scarcity problem [5], [17]–[20], or do not consider energy management [21]–[24]. Thus, they cannot fulfill the requirements of EHCRSNs. To fill this research gap, this paper proposes a framework to capture the dynamics of EH process and channel condition, and channel sensing inaccuracy. Based on the framework, a low-complexity
A. Sampling Rate and Utility

allocate the channels for network utility optimization. A joint algorithm is presented to jointly manage sensors’ energy and primary users and the sensor network.

Figure 1. An illustration of EHCRSN THAT shows the coexistence of the primary users and the sensor network.

algorithm is presented to jointly manage sensors’ energy and allocate the channels for network utility optimization.

III. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a single-hop EHCRSN of \( N \) sensors forming the set \( \mathcal{N} = \{1, 2, \ldots, N\} \) and operating over the time slots \( t \in \mathcal{T} = \{0, 1, 2, \ldots\} \). As shown in Fig. 1, the EHCRSN coexists with PUs that have the privilege to access licensed channels. The sensor collects data from an area of interest and saves it in its data queue, then transmits it to the sink over licensed channels. There are \( L \) transceivers mounted on the sink such that the sink can support \( L \) concurrent data transmission over \( L \) different frequency bands in each time slot. The availability information of the licensed spectrum is acquired from a third-party system (TPS). The TPS detects the PU activities by various existing spectrum-sensing technologies, such as energy detection [25].

Throughout this paper, we use the following notations. For a random variable \( X \), the expected value is denoted by \( \mathbb{E}[X] \), and its conditional expectation on event \( A \) is denoted by \( \mathbb{E}[X|A] \). The function \( [x]^+ \) denotes non-negative values, i.e., \( \max(x, 0) \).

A. Sampling Rate and Utility

In time slot \( t \), sensor \( n \) collects data at a sampling rate \( r_n(t) \), which falls in the range:

\[
0 \leq r_n(t) \leq r_{\text{max}}, \quad \forall n \in \mathcal{N},
\]

where \( r_{\text{max}} \) is the maximum sampling rate. The sampling rate is associated with a utility function \( U(r_n(t)) \), which is increasing, continuously differentiable and strictly concave in \( r_n(t) \) with a bounded first derivative \( U'(r_n(t)) \) and \( U(0) = 0 \) [26]. The concavity of the utility function is based on the observation that the marginal utility of the collected data decreases as the amount of collected data increases in sensor networks [5]. The upper bound of the first-order derivative of \( U(r_n(t)) \) is denoted by \( \zeta_{\text{max}} \) and equals \( U'(0) \).

B. Channel Detection and Allocation Model

The licensed spectrum is divided into \( K \) orthogonal channels of equal bandwidth. The set of orthogonal channels is denoted by \( \mathcal{K} = \{1, 2, \ldots, K\} \) with cardinality \( K = |\mathcal{K}| \). Let \( \mathbf{S}(t) = (S_1(t), \ldots, S_K(t)) \) denote the channel availability indicator with the interpretation that \( S_k(t) = 1 \) if channel \( k \) is available, and \( S_k(t) = 0 \) otherwise. We assume that the PU activity on channel \( k \) evolves following an independent and identical distribution (i.i.d.) across the time slots and is uncorrelated with sensors’ activities [21]. The channel unavailability rate which corresponds to the PU activity rate on channel \( k \) is given by \( \beta_k = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} (1 - S_k(t)) \leq 1 \).

The EHCRSN acquires the availability of channels at the beginning of each time slot from the TPS. Owing to detection errors of spectrum-sensing such as false-alarms and misdetection [27], the channel availability information is assumed to be imperfect. Thus, the TPS provides channel access probability vector \( \mathbf{P}(t) = (Pr_1(t), \ldots, Pr_K(t)) \), where \( Pr_k(t) \) denotes the probability that channel \( k \) is idle and hence accessible in time slot \( t \) [21]. Two factors impact the channel access probability: the actual PU activity on channel \( k \), i.e., \( S_k(t) \), and the accuracy of the spectrum-sensing techniques [25]. The performance of spectrum sensing techniques highly depends on the receiver signal-to-noise ratio (SNR) and the detection parameters (e.g., detection threshold) [27]. These conditions in the \( t^{th} \) time slot are collectively denoted by \( \Theta(t) \). The channel access probability \( Pr_k(t) \) is the conditional probability of the channel being available in time slot \( t \), i.e., \( Pr_k(t) = Pr[S_k(t) = 1|\Theta(t)] \) [21]. Because \( S_k(t) = 1 \) indicates that the availability of channel \( k \), with \( S_k(t) = 0 \) otherwise, the closer the value of \( Pr_k(t) \) is to that of \( S_k(t) \), the more accurate the channel availability information is. An EHCRSN with accurate \( Pr_k(t) \) is more efficient in utilizing the licensed channels by avoiding collisions.

At the beginning of each time slot, the sink allocates licensed channels to sensors based on the channel access probability. Let \( \mathbf{J}(t) \) denote the channel allocation matrix of elements \( J_{n,k}(t) \), \( \forall n \in \mathcal{N}, k \in \mathcal{K} \); \( J_{n,k}(t) = 1 \) if channel \( k \) is allocated to sensor \( n \), and otherwise is 0. To avoid interference among sensors, each channel can be allocated to one sensor at most,

\[
\sum_{n \in \mathcal{N}} J_{n,k}(t) \leq 1, \quad \forall k \in \mathcal{K}.
\]

Furthermore, each sensor can use at most one channel in each time slot, so we have

\[
\sum_{k \in \mathcal{K}} J_{n,k}(t) \leq 1, \quad \forall n \in \mathcal{N}.
\]

Because there are \( L \) transceivers mounted on the sink, the sink can support at most \( L \) concurrent data transmissions over licensed channels in each time slot. This can be written as,

\[
\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} J_{n,k}(t) \leq L.
\]

C. Collision Control Model

Due to the inaccuracy of channel availability and PU activities, PUs and sensors may collide over the channels. The EHCRSN may access the channel that is occupied by PUs, and thus both data transmissions from PUs and sensors
fail due to interference. We assume that the PU on channel $k$ can tolerate a time-average collision rate denoted by $\rho_k$ [21]. For example, $\rho_k = 1\%$ implies that the PU on channel $k$ can tolerate at most 1% of data loss. Recalling that the PU on channel $k$ is active with rate $\beta_k$, the target tolerable collision rate evaluates to $\beta_k \rho_k$. Define a collision indicator $C_k(t) \in \{0,1\}$. The collision indicator takes a value of 1 if collision occurs and is 0 otherwise. A collision occurs when an unavailable channel is allocated to one of the sensors, such that $C_k(t) = (1 - S_k(t)) \sum_{n \in N} J_{n,k}(t)$. The time-averaged rate of collision between PUs and sensors on the $k^{th}$ channel can be defined as

$$\bar{C}_k = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} C_k(t), \quad \forall k \in K.$$ 

$\bar{C}_k$ should be less than the target tolerable collision rate $\beta_k \rho_k$, i.e.,

$$\bar{C}_k \leq \beta_k \rho_k, \quad \forall k \in K. \quad (5)$$

To keep track of collisions between sensors and PUs, we define the virtual collision queue $Z_k(t)$ for each channel and a vector of virtual collision queues for all licensed channels, $\mathbf{Z}(t) = (Z_1(t), \cdots, Z_K(t))$. The collision queue occupancy is stable only if the time-average input rate $\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} C_k(\tau) = \rho_k \beta_k$ is less than the time-average service rate $\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} (1 - S_k(\tau)) = \bar{C}_k$, i.e.,

$$\lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} C_k(\tau) \leq \lim_{t \to \infty} \frac{1}{t} \sum_{\tau=0}^{t-1} (1 - S_k(\tau)), \quad \forall k \in K,$$

which is equivalent to the constraint (5). Therefore, stabilizing the collision queue for each channel maintains the required PU protection.

D. Energy Consumption Model

In each time slot, the $n^{th}$ sensor senses data with sampling rate $r_n(t)$ from the area of interest and saves it in the data queue. The energy consumption\(^1\) of data sensing is assumed to be a linear function of the sampling rate $r_n(t)$ [17] and denoted by $P_{s r_n}(t)$. If channel $k$ is allocated to the $n^{th}$ sensor, it transmits data to the sink with power $P_T$, $\forall n \in N$. Thus, the total energy consumption $P_n^{total}$ of the $n^{th}$ sensor in the $t^{th}$ time slot is

$$P_n^{total}(t) = P_{s r_n}(t) + \sum_{k \in K} J_{n,k}(t) P_T, \quad \forall n \in N.$$

Because the sampling rate $r_n(t)$ is bounded by $r_{max}$ and at most one channel can be allocated to a given sensor, i.e.,

\(^1\)The time is measured in unit size, thus the implicit multiplication by 1 slot is omitted when converting between power and energy [17] [18].
the dynamics of the data queue can be expressed as:

$$Q_n(t + 1) = Q_n(t) - \sum_{k \in K} J_{n,k}(t)x_n(t) + r_n(t), \quad (12)$$

where $J_{n,k}(t)x_n(t)$ captures the services process whereas $r_n(t)$ models the input process. This single-server queuing system is stable if the following network-stability constraint is satisfied [28]:

$$\lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} \mathbb{E}[Q_n(t)] < \infty. \quad (13)$$

Constraint (13) implies that the data queues of all sensors have finite time-average occupancy.

In a given time slot, the $n^{th}$ sensor can only transmit the available data in its queue; hence, the following data availability constraint must be satisfied in each time slot:

$$0 \leq x_n(t) \leq Q_n(t) \quad \forall n \in \mathcal{N}. \quad (14)$$

### G. Optimization Problem Formulation

Based on the aforementioned models, we formulate the stochastic optimization problem. The objective is to maximize the time-average aggregate network utility of EHCRSNs subject to the constraints mentioned above. The time-average aggregate network utility problem can be written as

$$\hat{O} = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} -\mathbb{E}[O(t)], \quad (15)$$

where $O(t) = \sum_{n \in \mathcal{N}} U(r_n(t))$ denotes the network utility in a time slot. To simplify the presentation, we use $r(t)$, $x(t)$ and $e(t)$ to denote the vectors of sampling rate $r_n(t)$, data transmission rate $x_n(t)$, and harvested energy $e_n(t)$ in time slot $t$, respectively. Additionally, let $\Gamma(t) \triangleq (r(t), e(t), x(t), J(t))$ represent the set of these variables in time slot $t$.

The network utility can be maximized by optimizing $\Gamma(t)$ under the following utility maximization formulation,

$$\text{UMP} \quad \max_{\Gamma(t)} \hat{O}$$

s.t. Eqs. (1) to (14)

In the following section, we decompose UMP into a series of deterministic subproblems and relax the collision constraint (5), network-stability constraint (13), and energy-availability constraint (8) by employing Lyapunov optimization.

### IV. PROPOSED FRAMEWORK

With the above-described structure of UMP, it is challenging to design a low-complexity online algorithm to optimize the aggregate network utility without a priori knowledge of the energy harvesting, PU activity and channel fading statistics. The proposed framework is developed on the basis of Lyapunov optimization under which the UMP problem is decomposed into three deterministic subproblems. This approach facilitates achieving a close-to-optimal aggregate network utility and stability, and does not require a priori knowledge of the above-mentioned stochastic processes statistics [28].

### A. Lyapunov optimization

We define the network state in time slot $t$ as $H(t) \triangleq (Z(t), Q(t), E(t), \Theta(t))$ which captures the occupancy of collision queue, data queue, and energy queue and the conditions that affect the accuracy of channel availability estimation. Define a Lyapunov function, $L(t)$, as the sum of squares of backlogs in the collision and data queues, and the spare capacity in sensors’ batteries as follows:

$$L(t) = \frac{1}{2} \sum_{k \in K} (Z_k(t))^2 + \frac{1}{2} \sum_{n \in \mathcal{N}} (Q_n(t))^2 + \frac{1}{2} \sum_{n \in \mathcal{N}} (-\hat{E}_n(t))^2, \quad (16)$$

where $\hat{E}_n(t) = \Omega - E_n(t)$ denotes the spare capacity of the $n^{th}$ sensor battery. The Lyapunov function $L(t)$ can be considered a scalar measure of the congestion in $Z_k(t)$ and $Q_n(t)$, and the capacity availability in sensors’ batteries. A small value of $L(t)$ indicates a low occupancy in the data and collision queues, as well as low spare capacity in energy queues $E_n(t)$, i.e., the batteries; the converse is also true. Additionally, we define the conditional Lyapunov drift as the one-slot difference of the Lyapunov function conditional on the network state, denoted by $\Delta(t) = \mathbb{E}[L(t+1) - L(t)|H(t)]$. The expectation is taken over the randomness of energy harvesting, PU activity and channel fading, as well as the randomness in the energy management and channel allocation actions.

By minimizing $\Delta(t)$ in each time slot, the data queue $Q_n(t)$ and collision queue $Z_k(t)$ are pushed towards zero to stabilize the data queues and collision queues such that the network-stability constraint (13) and tolerable collision constraint (5) can be satisfied. Furthermore, the energy queues $E_n(t)$ are pushed towards their capacity $\Omega$, such that sensors tend to recharge their batteries through energy harvesting. By carefully designing the value of $\Omega$, the energy queues are guaranteed to have enough energy for data sensing and data transmission such that the energy-availability constraint (8) can be satisfied. The value of $\Omega$ is determined in Theorem 2 in Section V. Thus, constraints (5), (8) and (13) are satisfied.

At this point, the network utility to be maximized has not yet been incorporated. Therefore, we include a weighted version of the network utility into the Lyapunov drift, and instead of minimizing $\Delta(t)$, we minimize the following drift-minus-utility $\Delta_V(t)$ function:

$$\Delta_V(t) \triangleq \mathbb{E}[(\Delta(t) - \lambda O(t))H(t)], \quad (17)$$

where $V$ is a non-negative importance weight that represents how much we emphasize on utility maximization [28]. In other words, instead of greedily minimizing $\Delta(t)$, we minimize $\Delta_V(t)$ to jointly stabilize the queues and optimize the weighted network utility $\lambda O(t)$. With a sufficiently large value of $V$, a close-to-optimal aggregate network utility can be achieved [29]. However, the data queues and energy queues become longer with a larger value of $V$, such that longer data queue buffers and battery capacities are required to support the EHCRSN. Thus, adjusting $V$ allows a trade-off between the reduction of queue length and optimization of the network utility.

Considering that drift-minus-utility $\Delta_V(t)$ is a quadratic
function of the queue lengths and variables in $\Gamma(t)$, Lemma 1 derives the upper bound of $\Delta_V(t)$. The upper bound is a linear function of the queue length and the variables in $\Gamma(t)$, which can be efficiently minimized.

**Lemma 1.** Given the variables in $\Gamma(t)$, the value of $\Delta_V(t)$ is upper-bounded by:

$$\Delta_V(t) \leq B + E[D_V(t) | H(t)], \tag{18}$$

where the value of constant $B$ is independent of $V$ and can be expressed as

$$B = \frac{N}{2} \left[ (\lambda_{max})^2 + (r_{max})^2 + (P_{max})^2 + (\eta_{max})^2 \right] + \frac{1}{2} [K + \sum_{k \in K} (\rho_k)^2], \tag{19}$$

and $D_V(t)$ is given in Eq. (20).

**Proof:** See Appendix A.

Rather than minimizing the drift-minus-utility $\Delta_V(t)$ function, we try to minimize its the upper bound, i.e., the right-hand side (RHS) of Eq. (18). Furthermore, for a given network condition $H(t)$, only $D_V(t)$ is relevant to the variables in $\Gamma(t)$. Therefore, we minimize $D_V(t)$ by solving for the optimal sampling rate $r(t)$, harvested energy $e(t)$, data transmission rate $x(t)$, and channel allocation $J(t)$ in each time slot.

**B. Framework Structure**

Exploiting the linear structure of Eq. (20), $D_V(t)$ can be minimized after being decomposed into three subproblems. In particular, the three subproblems are: battery management (BM), sampling rate control (SRC), and channel and data rate allocation (CDRA). Fig. 2 shows the three subproblems and the data flows among them. In the following, we treat each of the subproblems separately. The subproblems BM and SRC optimize the harvested energy $e_n(t)$ and sampling rate $r_n(t)$, respectively. Both BM and SRC require local information only available at the sensor, and they can be distributively solved at each sensor. However, CDRA is centrally solved at the sink because it requires information on the data queue occupancy $Q(t)$, energy queue occupancy $E(t)$, and channel collision queue occupancy $Z(t)$ of all sensors. The sink gathers this information at the beginning of each time slot via the common control channel, as in [30]. In the following, each of the subproblems is solved separately.

- **Battery Management**

  Considering the first term on the RHS of (20) and the relevant constraints (9) and (10), we have the following optimization problem to solve for $e_n(t)$

  $$(\text{BM}) \min_{e_n(t)} - \hat{E}_n(t)e_n(t)$$

  s.t. \(e_n(t) \leq \eta_n(t),\)

  \(E_n(t) + e_n(t) \leq \Omega.\)

  If the battery is not full, i.e., $E_n(t) < \Omega$ and $\hat{E}_n(t) > 0$, the sensor should harvest as much energy as possible. Hence, if $E_n(t) < \Omega$, we have

  \(e_n(t) = \min(\Omega - E_n(t), \eta_n(t));\) otherwise, \(e_n(t) = 0.\)

- **Sampling Rate Control**

  Considering the second term on the RHS of (20) with constraint (1), we have the following optimization problem to optimize the sampling rate $r_n(t)$:

  $$(\text{SRC}) \min_{r_n(t)} r_n(t)(Q_n(t) + P_S \hat{E}_n(t)) - VU(r_n(t))$$

  s.t. \(0 \leq r_n(t) \leq r_{max}.\)

  The utility function $U(r_n(t))$ is concave; thus, the SRC problem is convex. Let the sampling rate $r_n^*(t)$ be the optimal solution to the SRC problem, based on the convex optimization theory [31], we have:

  $r_n^*(t) = \left[ \frac{U^{-1} \left( \frac{Q_n(t) + P_S \hat{E}_n(t)}{V} \right)}{r_{max}} \right]_0^{r_{max}} \tag{21}$

  where $[z]_a^b = \min(\max(z, a), b)$ and $U^{-1}(\cdot)$ is the inverse of the first derivative of $U(\cdot)$.

- **Channel and Data Rate Allocation**

  Considering the third term on the RHS of (20) with constraints (2), (3), (4), (11) and (14), the problem of interest is determining the channel allocation matrix $J(t)$ and data transmission rate $x(t)$, which can be written as follows:

  $$(\text{CDRA}) \min_{J(t), x(t)} \sum_{n \in N} \sum_{k \in K} J_{n,k}(t) [Z_k(t)(1 - Pr_k(t)) - (Q_n(t)x_n(t)Pr_k(t) - P_T \hat{E}_n(t))]$$

  s.t. \(2)(3)(4)(11)(14).\)

  CDRA optimizes the data transmission rate $x(t)$ and channel allocation $J(t)$; the former is a continuous variable and the latter is an integer variable which makes
\[ D_V(t) = \sum_{n \in \mathcal{N}} \left[ -\hat{E}_n(t)e_n(t) \right] + \sum_{n \in \mathcal{N}} \left[ Q_n(t)r_n(t) + P_Sr_n(t)\hat{E}_n(t) - VU(r_n(t)) \right] \\
+ \sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} J_{n,k}(t) \left[ Z_k(t)(1 - Pr_k(t)) - (Q_n(t)x_n(t)Pr_k(t) - Pt\hat{E}_n(t)) \right] \]

(20)

This subproblem is a mixed integer problem. To facilitate the design of a tractable resource allocation solution, we transform CDRA into an integer problem by relaxing the constraints related to \( x(t) \), i.e., constraints (11) and (14). This is achieved over two steps. First, we adjust the data queue length and modify the objective function of CDRA; we refer to it hereafter as the modified CDRA (m-CDRA). We show that the objective function of m-CDRA is minimized if sensors transmit data at full capacity on their assigned channels. Second, we replace the continuous variable \( x_n(t) \) by the channel capacity \( \lambda_{n,k}(t) \) in the objective function of m-CDRA. Thus, we can relax constraints (11) and (14) and transform the m-CDRA problem into a Channel Allocation (CA) problem, which is a one-to-one matching problem. These steps are detailed in the following:

1) Define the adjusted length of the data queue as
\[
\hat{Q}_n(t) = \left[ Q_n(t) - \lambda_{max} \right]^+.
\]

(22)

After replacing \( Q_n(t) \) by \( \hat{Q}_n(t) \) in the objective function of the CDRA problem, we rewrite the objective function as
\[
\sum_{n \in \mathcal{N}} \sum_{k \in \mathcal{K}} J_{n,k}(t)M_{n,k}(t),
\]

(23)

where \( M_{n,k}(t) = [Z_k(t)(1 - Pr_k(t)) - (\hat{Q}_n(t)x_n(t)Pr_k(t) - Pt\hat{E}_n(t))] \). Instead of solving the original CDRA, we solve the modified m-CDRA with Eq. (23) as the objective function to find a suboptimal solution for the original CDRA. Suppose that \( J^*(t) \) and \( x^*(t) \) are the optimal solutions for the m-CDRA; in the following lemmas, we show that a channel \( k \) is assigned to sensor \( n \) if and only if it has a sufficient amount of data to transmit and it transmits it at full channel capacity.

**Lemma 2.** For a channel \( k \) to be assigned to sensor \( n \), i.e., \( \sum_{k \in \mathcal{K}} J_{n,k}^*(t) = 1 \), the following must be satisfied
\[
Q_n(t) > \lambda_{max}.
\]

(24)

**Proof:** See Appendix B.

**Lemma 3.** If any channel is assigned to the \( n^{th} \) sensor in the \( t^{th} \) time under the modified m-CDRA, i.e., \( \sum_{k \in \mathcal{K}} J_{n,k}^*(t) = 1 \), then we have
\[
x_n^*(t) = \sum_{k \in \mathcal{K}} J_{n,k}^*(t)\lambda_{n,k}(t),
\]

(25)

otherwise, \( x_n^*(t) = 0 \).

**Proof:** See Appendix C.

2) Lemma 3 shows that the sensor must fully utilize the assigned channel to optimally solve m-CDRA. Therefore, we can replace the transmission rate \( x_n(t) \) by channel capacity \( \lambda_{n,k}(t) \) in Eq. (23) and, thus, relax the channel capacity constraint (11) and data-availability constraint (14). The modified m-CDRA is transformed into a CA problem as follows:
\[
(CA) \min_{J(t)} \sum_{n,k} J_{n,k}(t) [Z_k(t)(1 - Pr_k(t)) - (\hat{Q}_n(t)x_n(t)Pr_k(t) - Pt\hat{E}_n(t))]
\]

s.t. (2)(3)(4).

CA can be mapped to a one-to-one matching problem. Furthermore, due to the limited number of transceivers on the sink, i.e., \( L \leq K \), a maximum of \( L \) channels can be allocated to sensors in a given time slot. Meanwhile, if \( L < K \), i.e., not all channels can be allocated to the sensors, CA is an unbalanced matching problem, which can be solved by the adaptive Hungarian algorithm proposed in [32]. The complexity of the algorithm increases linearly with the number of sensors.

C. Utility-optimal Resource Management and Allocation Algorithm (UoRMA)

In this subsection, we present the UoRMA algorithm in Algorithm 1. The UoRMA algorithm achieves the optimal harvested energy \( e^n(t) \), sampling rate \( r^n(t) \), data transmission rate \( x^n(t) \), and channel allocation \( J^n(t) \) by solving BM, SRC and CDRA, respectively. Moreover, the occupancy of data queues \( Q(t) \), energy queues \( E(t) \) and collision queues \( Z(t) \) are updated according to their respective queue dynamics.

Both the BM and SRC problems have closed-form solutions, and can be distributively solved at each sensor. Thus, their complexity is negligible. The complexity of Algorithm 1 is dominated by solving the CA problem in step 8 with time complexity of \( O(NKL + L^2\log(\min(N,K))) \) [32]. Therefore, the complexity of UoRMA increases linearly with the number of sensors \( N \). Notably, the complexity of algorithms designed based on Markov Decision Process (MDP) increases exponentially with \( N \) [33]. Comparing to the MDP-based algorithms, UoRMA is more computationally efficient in addition to being scalable for densely deployed sensor networks.

V. PERFORMANCE ANALYSIS

In this section, we analyze the stability and performance of the proposed UoRMA algorithm. Theorem 1 proves the stability of EHCRSNs operating under the UoRMA algorithm by deriving upper bounds on the length of the data queues and
collision queues. Then, we derive the required battery capacity to support the operation of the EHCRSN in Theorem 2. Theorem 3 evaluates the gap between the network's aggregate utility obtained by UoRMA and the optimal solution to demonstrate the optimality of UoRMA.

### A. Upper bounds on data queues and collision queues

We derive the upper bounds on the occupancies of queues and collision queues in Theorem 1. The existence of the bounds guarantees satisfying the data and collision queue stability constraints (13) and (5).

**Theorem 1.** For a non-negative parameter \( V \), \( P_k(t) \leq 1 - \epsilon \), \( \forall k, t \), and an initialization of the collision queue and data queue satisfying \( 0 \leq Z_k(0) \leq Z_{max} \), \( \forall k \in K \) and \( 0 \leq Q_n(0) \leq Q_{max} \), \( \forall n \in N \), the upper bounds are given by

\[
Q_{max} = \frac{\varsigma V + r_{max}}{P_{r}}, \\
Z_{max} = \frac{Q_{max} \lambda_{max}(1 - \epsilon)}{\epsilon} + 1,
\]

we have

\[
0 \leq Q_n(t) \leq Q_{max}, \quad \forall n \in N, \quad (26)
\]

\[
0 \leq Z_k(t) \leq Z_{max}, \quad \forall k \in K. \quad (27)
\]

**Proof:** See Appendix D.

As we can see from Eqs. (26) and (27), both the upper bounds of data queues and collision queues increase linearly with the weight \( V \). Since a larger \( V \) can bring higher network utility, the linear increase of upper bound on data queues indicates that a longer data buffer is required at each sensor to achieve better network performance. Furthermore, the increase of upper bound on collision queues also indicates that the PUs may experience more collisions from the EHCRSN. However, the collision constraint (5) can still be satisfied due to the existence of the upper bound on collision queues.

### B. Required battery capacity \( \Omega \)

In Theorem 2, we determine the required battery capacity \( \Omega \) in such a way that the sensor does not sense or transmit any data if the available energy is less than the maximum energy consumption of each sensor, i.e., \( E_n(t) \leq P_{max} \). Therefore, the energy-availability constraint (8) becomes implicit.

**Theorem 2.** Under the proposed framework and with a battery capacity \( \Omega \) given by

\[
\Omega = \max \left( \frac{V \varsigma U}{P_{S}} + P_{max}, \frac{Q_{max} \lambda_{max}}{P_{T}} + P_{max} \right), \quad \forall n \in N, \quad (28)
\]

sensor \( n \) does not sense data or is not allocated a channel, i.e., \( r_n(t) = 0 \) and \( \sum_{k \in K} J_{n,k}(t) = 0 \), if the energy queue length in a given time slot is less than the upper bound of the sensor's energy consumption, i.e., \( E_n(t) < P_{max} \).

**Proof:** See Appendix E.

The required battery capacity in (28) is determined by both the transmission power \( P_{T} \) and the sensing/processing power \( P_{S} \) because both data arrival and departure consume energy in EHCRSNs.

### C. Optimality of the UoRMA Algorithm

In Theorem 3, the optimality of the UoRMA algorithm is analyzed.

**Theorem 3.** Suppose that the optimal network utility that can be achieved by an exact and optimal algorithm is \( O^* \) and that the network utility \( \hat{O} \) achieved by the UoRMA algorithm satisfies:

\[
\hat{O} \geq O^* - \frac{\hat{B}}{V} \quad (29)
\]

where \( \hat{B} = B + NK(\lambda_{max})^2 \).

**Proof:** See Appendix F.

If we do not transform CDRA to CA, then the gap between the solution obtained by the proposed algorithm and the optimal solution can be determined by \( B/V \) [28], where \( B \) is the constant defined in Lemma 1. Thus, the performance loss caused by the transformation is shown in \( \hat{B} \), which is larger than \( B \). However, by Theorem 3, we see that the UoRMA algorithm can achieve an aggregate network utility within \( O(1/V) \) of the optimal utility without prior knowledge of the statistics of the stochastic processes such as channel fading, PU activities, and energy harvesting.
otherwise, \( \Pr(0.4 \text{ in each time slot}) = 0.1 \) [15]. The tolerable collision rate \( K \) in time slot \( t \) are empty at \( t = 0 \), such that sensors can sense data at \( t = 1 \).

Fig. 4 shows the data queue occupancy for different values of \( V \). The time-average lengths of data queues increase with the value of \( V \). Furthermore, it can be seen that the lengths of data queues converge quickly to the time-average value. This is because the battery is fully charged at \( t = 0 \), such that sensors can sense data at \( t = 1 \).

Fig. 5 shows the collision queue occupancy for different values of \( V \). Similar to the data queue dynamics shown in Fig. 4, the time-average lengths of the collision queues increase with larger values of \( V \), and the lengths of the collision queues fluctuate around a time-average value after the convergence. When the collision queue is small, the UoRMA algorithm tends to allocate the channel to sensors for data transmission. If the allocated channel is actually occupied by PUs, the collision queue increases back to the time-average value. Therefore, the collision queue length affects the dynamics of the queue’s fluctuation. In addition, sensors’ data queues and energy queues lengths also affect the dynamics of the fluctuation, because the UoRMA algorithm tends to allocate channels to the sensors with long data queues and small spare capacity in the energy queues.

**A. Network Utility and Queue Dynamics**

In Fig. 3, we evaluate the network utility versus the value of \( V \) ranging from 5 to 1200. The figure shows that the network utility increases with increase of \( V \). However, the rate at which the network utility increases decreases with larger \( V \). This is expected because the network utility is a concave function of \( V \), as shown in Eq. (42). We take a large value of \( V \) to illustrate the optimal network utility \( (V = 10^7 \text{ in our setting}) \). We compare the network utility obtained by \( V \) ranging from 5 to 1200 to the network utility obtained by \( V = 10^7 \). As shown in the figure, the increase of network utility from \( V = 1200 \) to \( V = 10^7 \) is quite limited in comparison to the increasing from \( V = 5 \) to \( V = 1200 \). Therefore, the network utility achieved when \( V = 1200 \) is close to the value of the optimal network utility.
B. Impact of Parameter Variations

In the following, we evaluate the impacts of various system parameters on the network utility. Assuming all channels have the same PU inactivity probability ranging from 0.5 to 0.9, we first verify the network utility in Fig. 6. The figure shows that the network utility increases with increase in the PU inactivity probability. At the same time, the rate of increase in the network utility decays with higher PU inactivity probability due to the limited energy supply rate.

However, similar to Fig. 6, the growth rate of the network utility decays with higher $\eta_{\text{max}}$. This indicates that, given sufficient energy supply, the network utility is bounded by the channel availability which also limits the sensors’ chance of transmitting data.

Fig. 8 shows the network utility versus transmission power $P_T$. As shown in the figure, there exists an optimal value of $P_T$ that maximizes the network utility. In our simulations, the optimal value of $P_T$ is 4. If $P_T$ is smaller than this optimal value, the available channels are underutilized which leads to lower network utility. However, if $P_T$ is larger than this optimal value, sensors need more time to harvest energy for data transmission, which also reduces the network utility.

Fig. 9 shows the network utility versus the number of transceivers that are mounted on the sink, $L$. Since the sink can support more concurrent data transmission with more transceivers, the network utility increases with $L$ when $L \leq K$, i.e., number of transceivers is not larger than the number of licensed channels.
VII. CONCLUSION

In this paper, we have developed an aggregate network utility optimization framework to facilitate the design of an online and low-complexity algorithm for managing and allocating the resources of EHCRSNs. The proposed framework captures and optimizes stochastic energy harvesting and consumption processes, as well as stochastic spectrum utilization and access processes. We employ Lyapunov optimization to decompose the problem into three sub-problems that are easier to solve, battery management, sampling rate control, and data rate and channel allocation. The solutions proposed to solve the three problems constitute the proposed utility-optimal resource management and allocation (UoRMA) algorithm. The optimality gap and bounds on data and energy queues are derived. The proposed algorithm achieves a close-to-optimal aggregate network utility while ensuring bounded energy and date queues. Simulations verify the optimality and stability of EHCRSN when operating under UoRMA algorithm. The outcomes of this work can be used to guide the design of practical EHCRSNs by guaranteeing PU protection and sensors sustainability.

For future work, we plan to investigate stochastic energy management and channel allocation for EHCRSNs to collect delay-sensitive data. In addition, the adaptive transmission power of sensors will be considered.

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APPENDIX A

PROOF OF LEMMA 1

By squaring both sides of Eq. (6), we have Eq. (30). Similarly, we have Eq. (31) from Eq. (12), and Eq. (32) from Eq. (7), respectively. Substituting $E[C_k(t)|\Theta(t)] = \sum_{k\in K} J_{n,k}(t) P_{R_k}(t)$ and $E[1 - S_k(t)|\Theta(t)] = 1 - P_{R_k}(t)$ into Eq. (31) and rearranging the equation, we have Eq. (20).

$$
\frac{1}{2} \left[ (Z_k(t+1))^2 - (Z_k(t))^2 \right] \\
\leq \frac{1}{2} \left[ (C_k(t))^2 + (\rho_k 1_k)^2 + 2Z_k(t)(C_k(t) - \rho_k 1_k) \right] \\
\leq \frac{1}{2} + (\rho_k)^2 + Z_k(t)(C_k(t) - \rho_k 1_k).
$$

Similarly, we have Eq. (31) from Eq. (12), and Eq. (32)

$$
\frac{1}{2} \left[ (Q_n(t+1))^2 - (Q_n(t))^2 \right] \\
\leq \frac{1}{2} \left[ \left( \sum_{k\in K} J_{n,k}(t) x_n(t) S_k(t) \right)^2 \right] \\
+ (r_n(t))^2 + 2Q_n(t) \left( r_n(t) - \sum_{k\in K} J_{n,k}(t) x_n(t) S_k(t) \right) \\
\leq (\lambda_{max})^2 + (r_{max})^2 + Q_n(t) \left( r_n(t) - \sum_{k\in K} J_{n,k}(t) x_n(t) S_k(t) \right)
$$

$$
\frac{1}{2} \left[ (E_n(t+1) - \Omega)^2 - (E_n(t) - \Omega)^2 \right] \\
\leq \frac{1}{2} \left[ (P_n(t))^2 + (e_n(t))^2 - 2E_n(t) (e_n(t) - P_n(t)) \right] \\
\leq \frac{(P_{max})^2 + (\eta_{max})^2 - 2E_n(t) (e_n(t) - P_{max})}{2}
$$

APPENDIX B

PROOF OF LEMMA 2

First, we prove that if there is any channel assigned to sensor $n$, its adjusted queue length $Q_n(t) > 0$. Suppose that channel $k$ is assigned to sensor $n$, i.e., $J_{n,k}(t) = 1$, it is obvious that $M_{n,k}(t) = Z_k(t)(1 - P_{R_k}(t)) + P_{R_k} E_n(t) - Q_n(t)x_n(t) P_{R_k}(t) < 0$. Since $Z_k(t)(1 - P_{R_k}(t)) \geq 0$, $P_{R_k} E_n(t) \geq 0$, and $x_n(t) P_{R_k}(t) \geq 0$, the adjusted data queue length becomes larger than zero, i.e., $Q_n(t) > 0$.

Then, we prove that if $\hat{Q}_n(t) > 0$, then the queue length $Q_n(t) > \lambda_{max}$. Recalling that $\hat{Q}_n(t) = \max(Q_n(t) - \lambda_{max}, 0)$, $\hat{Q}_n(t) > 0$ implies that $Q_n(t) - \lambda_{max} > 0$, i.e., the data queue length $Q_n(t)$ is larger than the maximum channel capacity $\lambda_{max}$.

APPENDIX C

PROOF OF LEMMA 3

We first consider the condition that sensor $n$ is not assigned with any channel, thus $J_{n,k} = 0, \forall k \in K$. According to constraint (11) and $x_n(t) \geq 0$, we have $x^*_n(t) = 0$.

Next, we prove that $x^*_n(t) = \sum_{k\in K} J^*_n(k) x^*_n(k)$ is the optimal solution for the condition that $\sum_{k\in K} J^*_n(k) = 1$. We use $k_n$ to denote the channel assignment to sensor $n$, i.e., $J^*_n(k_n) = 1$. Since $M_{n,k_n}$ is inversely correlated with the value of $x_n(t)$, the value of $x_n(t)$ should be as large as possible.
minimize $M_{n,k}$. The value of $x_n(t)$ is bounded by constraints (11) and (14), i.e., the channel capacity and data queue length $Q_n(t)$. According to Lemma 2, we can see that the data queue length $Q_n(t)$ must exceed the maximum channel capacity ($Q_n(t) \geq \lambda_{\text{max}}$) if $\sum_{k \in K} J_{n,k}(t) = 1$. Therefore, $x_n(t)$ is only bounded by channel capacity constraint in (11). Then we have the optimal $x_n(t)$ to be $x_n^*(t) = \lambda_{n,k_n}(t)$.

\section*{Appendix D
Proof of Theorem 1}

At $t = 0$, Eq. (26) holds. In the following, we prove Eq. (26) by inductions. We first assume that Eq. (26) holds in time slot $t$, and then prove that it holds in $t + 1$.

1) If sensor $n$ does not sense any data, then we have $Q_n(t + 1) \leq Q_n(t) \leq \zeta U + r_{\text{max}}$.

2) If sensor $n$ collects data with sampling rate $r_n^*(t)$, given in Eq. (21), then we have $U'U(r_n^*(t)) = Q_n(t) - P_S(E_n(t) - \Omega)$ and $Q_n(t) \leq U'U^*(r_n^*(t))$. Since $U'(r_n^*(t)) \leq \zeta U$, \forall $r_n(t)$ where $\zeta U$ denotes the upper bound of the first-order derivative of $U(r_n(t))$, \forall $r_n(t)$, we have $Q_n(t) \leq V \zeta U$. Furthermore, since $r_n^*(t) \leq r_{\text{max}}$, we have $Q_n(t + 1) \leq Q_n(t) + r_{\text{max}} \leq V \zeta U + r_{\text{max}}$.

Summarily, we have $Q_n(t + 1) \leq V \zeta U + r_{\text{max}}$. This completes the proof of Eq. (26).

Then we prove Eq. (27) by inductions. At $t = 0$, the collision queue is initialized as an empty queue. We prove that if Eq. (27) holds in time slot $t$, it will hold in $t + 1$.

1) If $P_k(t) = 1$, then no collision can happen, such that $Z_k(t + 1) \leq Z_k(t) \leq Z_{\text{max}}$.

2) If $P_k(t) \leq 1 - \varepsilon$, and $Z_k(t) \leq Z_{\text{max}} - 1$, then we have $Z_k(t + 1) \leq Z_k(t) + 1 \leq Z_{\text{max}}$.

3) If $P_k(t) \leq 1 - \varepsilon$, and $Z_k(t) > Z_{\text{max}} - 1$, then we have $Z_k(t) - P_k(1 - P_k(t)) - (E_n(t) - \Omega) - Q_n(t)x_n(t)P_k(t) \geq 0$, so channel $k$ cannot be allocated to any sensor in problem CA. This would yield $C_k(t) = 0$. Therefore, we have $Z_k(t + 1) \leq Z_k(t) \leq Z_{\text{max}}$.

Summarily, we have $Z_k(t + 1) \leq Z_{\text{max}}$. This completes the proof of Eq. (27).

\section*{Appendix E
Proof of Theorem 2}

We first derive an expression for $\Omega$ in such a way that sensor $n$ does not sense data, i.e., $r_n(t) = 0$ if $E_n(t) < P_{\text{max}}$. The sampling rate $r_n(t)$ is determined by Eq. (21). The utility function $U(r_n(t))$ is concave; therefore, $U'(r_n(t))$ and $r_n(t)$ are inversely proportional. Based on Eq. (21), sensor $n$ does not sense any data, i.e., the sampling rate is $r_n(t) = 0$, if

$$Q_n(t) + P_S \hat{E}_n(t) \geq \zeta U \geq U'(0).$$

Recall that $\hat{E}_n(t) = \Omega - E_n(t)$ and rearrange Eq. (33) to $\Omega \geq \frac{V \zeta U}{P_S} + P_{\text{max}}$. To satisfy the sensor cannot sense any data when $E_n(t) < P_{\text{max}}$, $\Omega$ can be set as follows $\Omega \geq \frac{V \zeta U}{P_S} + P_{\text{max}}$.

Then we derive the value of $\Omega$ in such a way that no channel can be allocated to sensor $n$, i.e., $\sum_{k \in K} J_{n,k}(t) = 0$, if $E_n(t) < P_{\text{max}}$. As we can see from the objective function of CA, no channel can be allocated to $n$ if

$$Z_k(t) + (1 - P_k(t)) + P_T \hat{E}_n(t) - Q_n(t) \lambda_{n,k}(t) P_k(t) \geq 0.$$  

Rearrange equation (34) to

$$\Omega \geq \frac{Q_n(t) \lambda_{n,k}(t) P_k(t) - Z_k(t)(1 - P_k(t)) + E_n(t)}{P_T}.$$  

Since $P_k \leq 1$, $\hat{Q}_n(t) \leq Q_{\text{max}}, Z_k(t) \geq 0$ and $\lambda_{n,k}(t) \leq \lambda_{\text{max}}$, we can change the RHS of Eq. (35) to $Q_{\text{max}} \lambda_{\text{max}} / P_T + E_n(t)$. To guarantee that no channel can be allocated to sensor $n$ if $E_n(t) < P_{\text{max}}$, $\Omega$ can be set to $\Omega \geq \frac{Q_{\text{max}} \lambda_{\text{max}} / P_T}{P_{\text{max}}}$. Theorem 2 is thus proved.

\section*{Appendix F
Proof of Theorem 3}

We prove the theorem by comparing the Lyapunov drift with a stationary and randomized algorithm denoted by $\Pi$. We introduce superscript $\Pi$ to variables $r^\Pi(t), \psi^\Pi(t), J_t^\Pi(t)$ and $p_t^\Pi(t)$ to indicate that these variables are generated under algorithm $\Pi$. Since all of the PU activities, channel condition, and EH process change in i.i.d manners across the time slots, according to Theorem 4.5 in [28], algorithm $\Pi$ can yield

$$E \left[ \sum_{n \in N} U(r_n^\Pi(t)) \right] \leq O^* + \delta,$$

$$E \left[ \sum_{k \in K} \left( C_k^\Pi(t) - \rho_k(1 - S_k(t)) \right) \right] \leq \rho_1 \delta,$$

$$E \left[ \sum_{n \in N} \left( r_n(t) - \sum_{k \in K} J_n^\Pi(t)x_n(t)S_k(t) \right) \right] \leq \rho_2 \delta,$$

$$E \left[ \sum_{n \in N} \left( \psi_n^\Pi(t) - P_{\text{max}} \right) \right] \leq \rho_3 \delta,$$

where $\delta \geq 0$ can be arbitrarily small, and $\rho_1, \rho_2$ and $\rho_3$ are constant scalars.

In each time slot, the UoRAM algorithm minimizes the right hand side of the Lyapunov drift in Eq. (40)

The proof of Eq. (40) can be obtained by Theorem 2 in [18]. Note that $\Delta(t) = \sum_{n \in N} U(r_n(t)) \leq B + E[\tilde{D}_V(t)]H(t), \text{ where } B = B + NK(\lambda_{\text{max}}^2)$ is a constant w.r.t. the variables, we can have the following inequality:

$$\Delta(t) - (B + E[\tilde{D}_V^{\Omega_R}(t)]H(t)) \leq \tilde{B} + E[\tilde{D}_V^{\Omega_R}(t)]H(t) \leq \tilde{B} + E[\tilde{D}_V^{\Pi}(t)]H(t) \leq \tilde{B} + (\rho_1 + \rho_2 + \rho_3) \delta + O^* + \delta,$$

where $\tilde{D}_V^{\Omega_R}(t)$ and $\tilde{D}_V^{\Pi}(t)$ denote the value of $\tilde{D}_V(t)$ obtained under UoRAM algorithm and algorithm $\Pi$, respectively.
respectively. By setting $\delta$ to zero, we can have
\[
\Delta(t) - V E \left[ \sum_{n \in \mathcal{N}} U(r_n(t)) \right] \leq O^* + \bar{B}. \tag{42}
\]
Taking the expectation on both sides of (42), summing up the equations for $t \in T$, dividing by $T$ and letting $T \to \infty$, we have $O \geq O^* - \bar{B}/V$. Theorem 2 is thus proved.

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