This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the ICC 2008 proceedings.

OFDMA Based Two-hop Cooperative Relay Network Resources Allocation

Mohamad Khattar Awad, *Student Member*, *IEEE*, and Xuemin (Sherman) Shen, *Senior Member*, *IEEE*
Department of Electrical & Computer Engineering,
University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada
mohamad@ieee.org, xshen@bbcr.uwaterloo.ca

Abstract— **In this paper, we focus on the resources allocation for the OFDMA based two-hop relay network which consists of a single base station, dedicated fixed relay stations and subscriber stations. Subscriber stations are allocated the subcarriers and relay stations that are required to satisfy their minimum rate requirements in either non-cooperative mode (i.e., direct communication with the base station) or in cooperative mode with one of the available relay stations. The cooperation is limited to one relay station to reduce the complexity incurred by the need for synchronization with multiple relays and with the base station at the PHY layer. The subcarriers and relay stations allocation problem is formulated as a Binary Integer Programming (BIP) problem with QoS constraints (minimum rate) and a practical synchronization constraint (cooperation with a single relay). Since the formulated problem is NP-complete, a simple sub-optimal algorithm is proposed to manage the multi-service network resources. Simulations and complexity analysis show that the presented algorithm achieves a network near optimal resources allocation with low computational complexity.**

I. INTRODUCTION AND RELATED WORK

The broadband wireless access (BWA) networks is emerging to replace the wireline DSL and T1 line networks [1]. In spite of their low deployment cost compared to wireline networks, wireless wireless channel impairments. In particular, the channel suffers from frequency selective fading and distance dependent fading (i.e., large-scale fading). While frequency selective fading results in inter-symbol-interference (ISI), large-scale fading attenuates the transmitted signal below a level at which it can be correctly decoded. The advanced PHY layer technology, Orthogonal Frequency Division Multiple Access (OFDMA), eliminates the frequency selectivity effect by transmitting the wide-band signal on multiple orthogonal subcarriers as narrow-band signals. Hence, the channel frequency selectivity (frequency nulls) affects only selected narrow-band signals. OFDMA exploits multi-user diversity in frequency selective channels by exclusively assigning a subset of subcarriers to the subscriber station with the highest channel gain [2]. In the Amplify-and-Forward scheme (AF), relaying overcomes large-scale fading by amplifying the received signal then re-transmitting it to the receiver which translates to higher transmission rates and spectral efficiency [3].

To further exploit the wireless channel capacity, a relay station can cooperate with a subscriber station in the Time Division Duplex (TDD) scheme. For this scheme, the receiver receives the direct signal from the transmitter in the first half of the time slot and receives the same signal but amplified by the relay station in the second half of the time slot [4], [5]. The OFDMA essential property in eliminating frequency selective fading and the relaying effectiveness in compacting large-scale fading in addition to cooperative relaying effectiveness in enhancing transmission rates motivate the integration of these technologies into one network architecture. Wireless networks of such architecture are expected to provide high-speed broadband service that provides QoS guarantees on the data, voice and video traffic. One direct implementation of this architecture is the multihop relay BWA networks currently under standardization by the IEEE 802.16j task group [6]. Similar to the original standard IEEE 802.16 [7], the amendment IEEE 802.16j is expected to adopt OFDMA with adaptive modulation and coding (AMC) at the PHY layer. The problem of resource allocation in this network is NP-Complete, and an optimal resource allocation can not be obtained in a short time comparable to the channel coherence time. Further, the computation burden increases as the number of subscriber stations, relay stations and subcarriers increase especially when QoS constraints and practical synchronization constraint are imposed. An efficient and low complexity resource allocation algorithm is needed.

A resource allocation protocol that allocates subcarriers to cooperating subscriber and relay stations was proposed in [8]. The QoS constraint is considered to be the fair utilization of the relaying nodes (i.e., each relay station relays signals on a limited number of subcarriers), an important assumption in sensor networks where relaying nodes are very limited in power, but not the system model under consideration. [9] presents a centralized heuristic algorithm to allocate power and subcarriers to user nodes and relays in a network where the node can establish a connection either through a direct connection or through the one relay but not in cooperative mode. Research activities in [8], [9] tend to focus on maximizing the total network throughput rather than satisfying each user's required rate.

In this paper, we propose a low complexity resource allocation protocol for OFDMA based two-hop relay networks under the following constraints: (i) Each subscriber station communicates with the base station either in non-cooperative mode or in cooperative mode with only one of the available relay stations; (ii) Users are allocated a sufficient number of subcarriers to guarantee their minimum rate requirements; (iii) Each subcarrier is exclusively allocated to one subscriber and relay station pair. The proposed algorithm is efficient in

978-1-4244-2075-9/08/$25.00 ©2008 IEEE

utilizing the available subcarriers and relay stations and robust in terms of computational complexity.

The reminder of the paper is organized as follows. Section II introduces a system model of the OFDMA relay network under consideration. Problem formulation is presented in section III. The resource allocation algorithm as well as the complexity analysis are presented in section IV. Performance of the algorithm is analyzed via simulations in section V, followed by the conclusions in section VI.

II. SYSTEM MODEL

Consider a single cell scenario with one base station at the center of the cell, multiple fixed relay stations, and multiple subscriber stations. Full channel state information (CSI) is assumed to be available to the stations using literature available channel estimation methods, e.g., [10]. Subscriber stations and relay stations report their CSI to the base station. In addition, subscriber stations report their minimum rate requirements to the base station. The central resource allocation unit at the base station performs the resource allocation, then reports the subcarriers assignments and relay stations assignments to each subscriber station and relay station.

The relay stations are assumed to be fixed on a circle with a radius equal to half of the cell radius in order to eliminate the effect of relay stations placements. In other words, the cell coverage is equally supported by the available relay stations (Fig. 3). The relay station forwards the received signal to the subscriber station by employing the Amplify-and-Forward (AF) forwarding scheme on the same subcarrier [5], [11]. We consider a TDD transmission pattern that is half-duplex in a sense that the subscriber station transmits while the relay station and base station receive in the first half of the time slot. In the second half of the slot, the relay station transmits to the base station. Fig. 1 illustrates both cooperative and non-cooperative TDD transmission patterns.

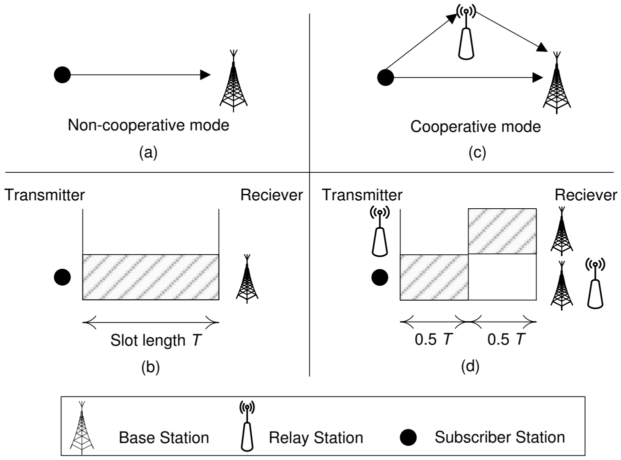

Fig. 1. The TDD scheme in both cooperative and non-cooperative modes.

The maximum allowable transmission power is assumed to be uniformly distributed among the source and relay stations [5]. Adaptive modulation and coding is employed to facilitate the higher channel gain achieved by OFDMA via exploiting multi-user diversity. Because subscriber stations and relay stations are fixed and their channels experience slow fading, the channel is assumed to be static over the resource allocation duration [8], [12], and is assumed to be frequency selective with a coherence bandwidth greater than the transmitted narrow-band signals' bandwidth.

III. PROBLEM FORMULATION

Fig. 2 depicts the network under consideration. There are A subscriber stations forming the set $\mathcal{A} = \{s_1, \cdots, s_a, \cdots, s_A\}$. The available B relay stations forms the set $\mathcal{B} = \{r_1, \cdots, r_b, \cdots, r_B\}$. The destination base station is symbolized by d. A subscriber stations share a total of N_{sc} subcarriers available to the cell. The set of subcarriers is denoted by $\mathcal{N} = \{1, \cdots, n, \cdots, N_{sc}\}$. As mentioned in the system model, the subscriber stations can establish a connection with d either in non-cooperative mode or in cooperative mode (i.e., with the cooperation of one relay station $r_b \in \mathcal{B}$) by employing the AF scheme.

The achievable rate of the cooperative AF protocol, shown in Fig. 1-(c) and (d) is analyzed in [13]. The maximum achievable rate in (bits/sec/Hz) by s_a on subcarrier n with the cooperation of r_b is given by:

$$I_n^{ab} = \frac{1}{2}\log_2\left(1 + \frac{\gamma_n^{ad}}{N_0} + \frac{|\beta_n^b|^2|\gamma_n^{ab}|^2|\gamma_n^{bd}|^2}{\left(|\beta_n^b|^2|\gamma_n^{bd}|^2 + 1\right)N_0}\right). \qquad (1)$$

where $|\gamma_n^{ad}|^2$,$|\gamma_n^{ab}|^2$ and $|\gamma_n^{bd}|^2$, respectively, are the nth subcarrier SNR from s_a to d, s_a to r_b and r_b to d. β^b is the relay r_b's amplifying gain. The direct transmission maximum achievable rate in (bits/sec/Hz) over both time slots as illustrated in Fig. 1-(a) and (b) is given by the well known result:

$$I_n^{ad} = \log_2\left(1 + \frac{\gamma_n^{ad}}{N_0}\right). \qquad (2)$$

We formulate the resources allocation problem as a BIP to maximize the network achievable rate while satisfying subscribers' minimum rate requirements $\overline{c^a}$ and limiting cooperation to one relay station. Let $y^{ab} \in \{0, 1\}$, where $y^{ab} = 1$ means that s_a is cooperating with r_b, whereas $y^{ab} = 0$ means s_a is not cooperating with $r - b$. Also, let $x_n^{ab} \in \{0, 1\}$ where $x_n^{ab} = 1$ means that the subcarrier n is allocated to the pair $s_a - r_b$ and $x_n^{ab} = 0$ otherwise. Because both the subscriber station and relay station transmit on the same frequency, but in different time slots, and the resource allocation is time independent, the base station d can be represented by a virtual relay r_{B+1}. Adding a virtual relay station enlarges the set $\mathcal{B}$ to $\mathcal{B}^+$. To use a uniform cost function in the optimization problem, we combine equations (1) and (2) in the following:

$$c_n^{ab} = [1 - \delta(b - (B+1))]I_n^{ab} + \frac{1}{2}\delta(b - (B+1))I_n^{ab} \qquad (3)$$

where $\delta(\cdot)$ is the Dirac delta function, $\beta_n^{(B+1)} = 0$ and $d =$

$B+1$. Mathematically, The optimization problem is

$$
\begin{array}{llll}
\max\limits_{x_n^{ab},y^{ab}} & \sum\limits_{a=1}^{A}\sum\limits_{b=1}^{B+1}\sum\limits_{n=1}^{N_{sc}} x_n^{ab}c_n^{ab} & & (4)\\
\text{s.t.} & \sum\limits_{b=1}^{B+1} y^{ab}=1 & \forall a & (5)\\
 & x_n^{ab}\le y^{ab} & \forall a\ \forall b\ \forall n & (6)\\
 & \sum\limits_{a=1}^{A}\sum\limits_{b=1}^{B+1} x_n^{ab}\le 1 & \forall n & (7)\\
 & \sum\limits_{n=1}^{N_{sc}}\sum\limits_{b=1}^{B+1} x_n^{ab}c_n^{ab}\ge \overline{c^a} & \forall a & (8)\\
 & x_n^{ab},\ y^{ab}\in\{0,1\} & & (9)
\end{array}
$$

Constraints (5) and (6) are limitations on the number of relay stations cooperating with each subscriber station to be one. Constraint (7) satisfies the definition of OFDMA that each subcarrier is allocated to one $s_a - r_b$ pair [2]. The constraint (8) guarantees that subscriber stations' minimum rate requirements $\overline{c^a}$ are met. An admission controller is assumed to be in place such that the problem is feasible and constraint (8) can be satisfied. The resources management problem is a BIP problem which is proven NP-complete [14], thus, it is intractable. However, an optimum solution can be obtained using optimization tools (e.g., CPLEX[1]) which suffer long running time that makes it impractical. In addition, optimization tools can only solve a problem of limited size, thereby motivation the development of a resource allocation algorithm that obtains near optimum solution in a considerably shorter time, less than the channel coherence time.

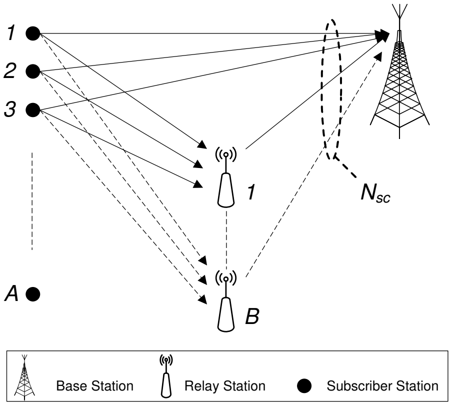

Fig. 2. The OFDMA based two hop relay network.

IV. PROPOSED ALGORITHM AND COMPLEXITY ANALYSIS

The proposed algorithm is greedy in a sense that a subcarrier is allocated to the subscriber station that achieves the maximum rate on it either with the cooperation of one of the available relays or directly in the current iteration without considering the future impact of the assignment decision [15]. Once a subscriber is cooperating with one relay station, it is allocated the subcarrier(s) with maximum achievable rate on the same relay station until its rate requirement is satisfied. In frequency selective channels, if the subscriber experiences high gain on a subcarrier, it is possible that there are other subcarriers with a high gain. In other words, in a closely packed spectrum, neighboring subcarriers experience similar gain. After all subscriber stations achieve their required rates, the remaining subcarriers are allocated to subscribers that achieve the maximum rate on them. $\mathcal{N}_a$ is the set of subcarriers assigned to subscriber s_a. (a,b) is a pair of cooperating subscriber station and relay station. The pairs (a,b) form the set $\mathcal{P}$. The total achievable rate by subscriber s_a is denoted by c^a. The proposed greedy algorithm is outlined below:

Algorithm 1 Greedy Algorithm

```
Satisfy subscribers' rate requirements
while $\mathcal{A} \neq \emptyset$ do
    $n \leftarrow random\,(\mathcal{N})$
    $(a^*, b^*) = \arg\max_{a\in\mathcal{A}\ b\in\mathcal{B}^+} c_n^{ab}$
    $\mathcal{N}_{a^*} \leftarrow \mathcal{N}_{a^*} \cup \{n\}$   $\mathcal{N} \leftarrow \mathcal{N} \setminus \{n\}$
    $c^{a^*} \leftarrow c^{a^*} + c_n^{a^*b^*}$
    while $c^{a^*} < \overline{c^{a^*}}$ do
        $n^* = \arg\max_{n} c_n^{a^*b^*}$
        $\mathcal{N}_{a^*} \leftarrow \mathcal{N}_{a^*} \cup \{n^*\}$   $\mathcal{N} \leftarrow \mathcal{N} \setminus \{n^*\}$
        $c^{a^*} \leftarrow c^{a^*} + c_{n^*}^{a^*b^*}$   $\mathcal{P} \leftarrow \mathcal{P} \cup \{(a^*, b^*)\}$
    end while
    $\mathcal{N} \leftarrow \mathcal{N} \setminus \mathcal{N}_{a^*}$   $\mathcal{A} \leftarrow \mathcal{A} \setminus \{a^*\}$
end while
Allocate remaining subcarriers
while $\mathcal{N} \neq \emptyset$ do
    $(a^*, b^*) = \arg\max_{(a,b)\in\mathcal{P}} c_n^{ab}$
    $\mathcal{N}_{a^*} \leftarrow \mathcal{N}_{a^*} \cup \{n\}$   $\mathcal{N} \leftarrow \mathcal{N} \setminus \{n\}$
end while
```

To analyze the computational complexity, we study its complexity in terms of the number subscriber stations A, relay stations $B+1$ and subcarriers N_{sc}. To satisfy the subscribers' rate requirements, the outer loop iterates A times. For a randomly selected subcarrier, finding the subscriber-relay stations pair that achieves the maximum rate requires $\mathcal{O}(AB)$ comparisons. The inner while loop iterates $N-1$, in the worst case and each iteration requires $N-1$ comparisons; hence, it is $\mathcal{O}(N^2)$. Allocating the remaining subcarriers requires, at most, $\mathcal{O}(N_{sc} - A)$ iterations and $\mathcal{O}(A)$ comparisons. Hence, the total algorithm complexity is $\mathcal{O}(A^2B + AN_{sc}^2)$. The proposed algorithm is low in computational complexity compared to the complexity required for the complete search over the problem space that is ABN_{sc}.

V. PERFORMANCE EVALUATION

Performance evaluations focus on the following questions: (1) How does the proposed algorithm allocation compare to the

---

[1] A Mathematical Programming Optimizer, www.ilog.com.

optimal allocation obtained by the optimization tool in terms of achieving the total network rate and running time? (2) Does the algorithm guarantee the subscribers' rate requirements? (3) How does the algorithm perform as subscribers' traffic intensity increases?

The simulated network consists of a base station located at the center of a cell with a 300m radius. Relay stations are placed on a circle with a 150m radius at equal angular distances. Subscriber stations are uniformly distributed in the cell coverage area. Fig. 3 shows a snapshot of the simulated network for $A = 50$ and $B = 6$. In this scenario, the cell coverage area is equally shared by the available relay stations.

The wireless channels are simulated to experience both frequency selective and large-scale fading. The subscriber stations and relay stations receive three Rayleigh distributed signals. The received signals' real and imaginary components for different subscribers are generated from an uncorrelated multidimensional Gaussian distribution with zero mean and the identity covariance matrix. Uncorrelated multi-path components lead to uncorrelated subscribers' frequency responses in the frequency domain. Thus, full multiuser diversity can be exploited. However, neighboring subcarriers gains within each subscriber frequency response are correlated. The large-scale fading is distance dependant and follows the inverse-power law:

$$|\gamma_n|^2 = D^{-\kappa} \cdot |\alpha_n|^2. \tag{10}$$

where D is the distance between the transmitter and receiver in meters, κ is the path loss exponent, and $|\alpha_n|^2$ is the nth subcarrier channel gain at the transmitter. In our simulation, the path loss exponent is set to 2, $\kappa = 2$.

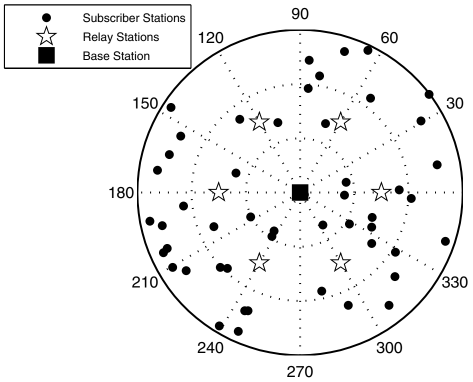

Fig. 3. A representative cell with 50 subscribers and 6 relays.

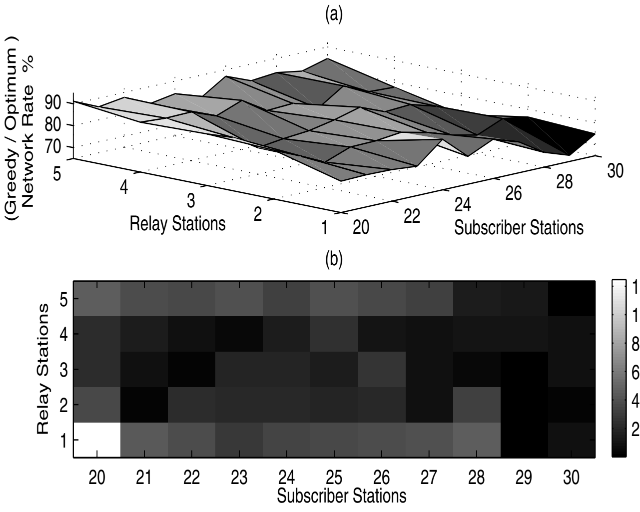

Fig. 4. (a) The ratio of the proposed algorithm network maximum achievable rate to optimal achievable rate. (b) The ratio of the proposed algorithm running time to optimization tool running time. For a network with $N_{sc} = 64$.

Fig.4-(a) shows the ratio of the network maximum achievable rate obtained by the proposed algorithm to the optimal network achievable rate obtained by the optimization package. The optimization package used in the simulation is CPLEX that solves for the optimal solution by employing branch-and-bound techniques [16]. To evaluate the algorithm performance for various sizes of the problem, we vary the number of available relay stations from 1 to 5 and the number of subscriber stations from 20 to 30 in a cell with 64 subcarriers ($N_{sc} = 64$). It is clearly observed that the proposed algorithm achieves 70% to 90% of the optimum. However, the required running time is in the range of only 2% to 12% of the optimization tool running time.

A network with 64 subcarriers ($N_{sc} = 64$) can support up to 25 subscriber stations ($A = 25$) assuming that each subscriber station needs 2 subcarriers at most ($\mathcal{N}_a \leq 2$) to satisfy its minimum rate requirements. In the worst case where the number of subscriber stations is as large as possible, the proposed algorithm achieves 70% of the optimum in only 2% of the running time required by the optimization tool (Fig. 4). For a smaller problem size that consists of $A = 20, B = 1$ and $N_{sc} = 64$, the optimization tool requires an additional 78% running time to achieve an optimal solution that is only 13% larger than the sub-optimal solution. Therefore, the proposed algorithm achieves a near optimal solution in a relatively short time.

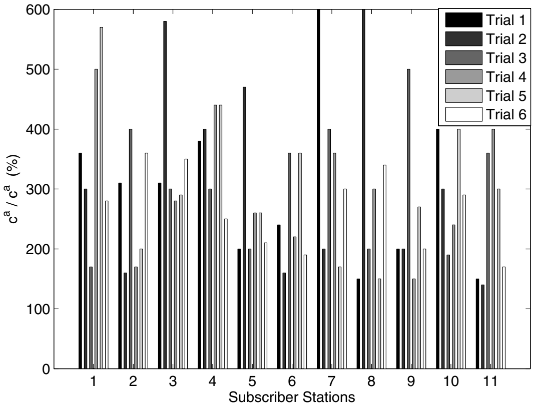

Fig. 5. The ratio of the allocated rate to minimum required rate.

In multi-service networks, minimum rate requirements are user specific. The proposed algorithm performance in satisfying subscriber stations' minimum rate $\overline{c_a}$ is evaluated in Fig. 5. Subscribers' $\overline{c^a}$ are randomly generated in each trial. Fig. 5, represents the ratio of each subscriber station's average assigned rate to the minimum required rate ($c^a/\overline{c^a}$) over six randomly generated subscriber stations' rate requirements (i.e., $c^a/\overline{c^a} \geq 100\%$). The analysis shows that all subscriber stations's rates are satisfied. Some subscriber stations are allocated more rates than their requirements, in order to maximize the total network throughput. Subscriber stations close to relay stations or the base station may experience higher channel gain from others. Thus, allocating the remaining subcarriers to these subscriber stations maximizes the network achievable rate.

Whereas the previously mentioned simulations evaluate the proposed algorithm efficiency and robustness in achieving a near optimal allocation, we further demonstrate the subscriber stations' transmission performance. Consider a network of 25 subscriber stations ($A = 25$), 4 relay stations ($B = 4$) and 64 subcarriers ($N_{sc} = 64$). Subscriber stations' packet arrival follows the Poisson process with an equal normalized average arrival rate of 0.4 packet/sec. The *Dropping rate* is defined as the ratio of dropped packets to arrived packets in a time slot. Packets are dropped when the buffer size reaches the threshold. *Throughput* is defined as the number of successfully transmitted packets in a time unit. The normalized throughput and normalized dropping rate average performance are achieved over 1000 simulation runs. Fig. 6 shows that the proposed algorithm satisfies subscriber stations' normalized rates. It is observed from Fig. 6-(a) that, on average, the normalized dropping rate is zero for arrival rates less than $\overline{c_a} = 0.4$. Similarly, Fig. 6-(b) shows that the proposed algorithm guarantees throughput greater than or equal the required rate. However, as the arrival rate increases, subscriber stations share the available network resources.

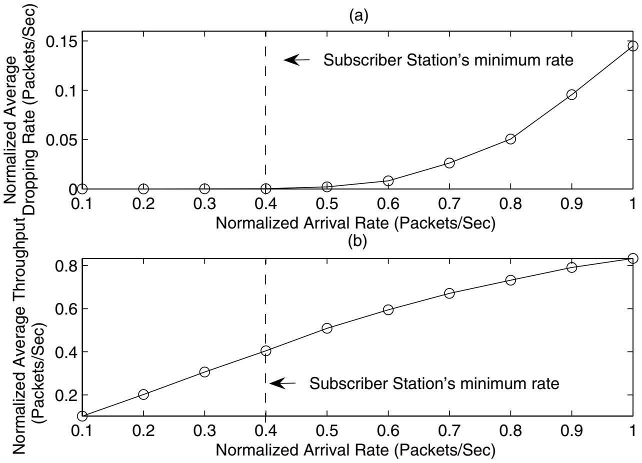

Fig. 6. (a) The average per user normalized average dropping rate. (b) The average per user normalized average throughput.

VI. CONCLUSIONS

In this paper, the resource allocation for the two-hop OFDMA cooperative relay networks has been addressed. Given the full channel state information among communicating stations, the algorithm at the base station allocates subcarriers and relay stations to the subscriber stations while satisfying their minimum rate requirements. Numerical and complexity analysis demonstrate that the proposed algorithm achieves near optimal allocation in relatively short running time. In particular, the algorithm achieves about 70% to 90% of the optimum in only 2% to 12% of the optimization tool running time. Since the proposed algorithm is low in complexity given the channel state information, our future wok is to develop a channel estimation error resilient resource allocation algorithm.

VII. ACKNOWLEDGMENTS

The authors would like to thank Professor Jochen Könemann from the Department of Combinatorics and Optimization at the University of Waterloo for his helpful comments and suggestions.

REFERENCES

[1] Z. Abichar, Y. Peng, and J. Chang, "Wimax: The emergence of wireless broadband," *IEEE IT Professional*, vol. 8, no. 4, pp. 44–48, 2005.

[2] H. Liu and G. Li, *OFDM-Based Broadband Wireless Networks: Design and Optimization*. Hobken, New Jersey: Wiley-Interscience, 2005.

[3] R. Pabst, B. Walke, D. Schultz, P. Herhold, H. Yanikomeroglu, S. Mukherjee, H. Viswanathan, M. Lott, W. Zirwas, M. Dohler, H. Aghvami, D. Falconer, and G. Fettweis, "Relay-based deployment concepts for wireless and mobile broadband radio," *IEEE Communications Magazine*, vol. 42, no. 9, pp. 80–89, Sept. 2004.

[4] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[5] J. Cai, X. Shen, J. W. Mark, and A. S. Alfa, "Resource allocation in wireless relay networks," in *IEEE Global Telecommunications Conference, GLOBECOM '06*, 2006, pp. 1–5.

[6] "Draft standard for local and metropolitan area networks–part 16: Air interface for fixed and mobile broadband wireless access systems–multihop relay specification," *IEEE Unapproved Draft Std P802.16j/D1*, 2007.

[7] "IEEE standard for local and metropolitan area networks part 16: Air interface for fixed broadband wireless access systems," *IEEE Std 802.16-2004*, pp. 1–857, 2004.

[8] G. Li and H. Liu, "Resource allocation for OFDMA relay networks with fairness constraints," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 11, pp. 2061–2069, 2006.

[9] L. Huang, M. Rong, L. Wang, Y. Xue, and E. Schulz, "Resource allocation for OFDMA based relay enhanced cellular networks," in *IEEE 65th Vehicular Technology Conference. VTC '07-Spring.*, 2007, pp. 3160–3164.

[10] J. Cai, X. Shen, and J. Mark, "Robust channel estimation for OFDM wireless communication systems - an H_∞ approach," *IEEE Transactions on Wireless Communications*, vol. 3, no. 6, pp. 2060–2071, 2004.

[11] N. Esseling, B. Walke, and R. Pabst, "Performance evaluation of a fixed relay concept for next generation wireless systems," in *15th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications. PIMRC '04*, vol. 2, 2004, pp. 744–751.

[12] T. Ng and W. Yu, "Joint optimization of relay strategies and resource allocations in cooperative cellular networks," *IEEE Journal on Selected Areas in Communications*, vol. 25, no. 2, pp. 328–339, 2007.

[13] G. Li and H. Liu, "On the capacity of broadband relay networks," in *Conference Record of the Thirty-Eighth Asilomar Conference on Signals, Systems and Computers.*, vol. 2, Nov. 2004, pp. 1318 – 1322.

[14] L. Wolsey, *Integer programming*. New York, NY: John Wiley & Sons Inc., 1998.

[15] T. Cormen, C. E. Leiserson, and R. L. Rivest, *Introduction to Algorithms*. Cambridge, Massachusetts: MIT Press, 2001.

[16] ILOG Inc., "ILOG CPLEX," Sept. 2007. [Online]. Available: http://www.ilog.com/products/cplex/