Uplink Ergodic Mutual Information of OFDMA-based Two-hop Cooperative Relay Networks with Imperfect CSI

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***Abstract*— This paper analyzes the ergodic mutual information of OFDMA cooperative relay networks when the Channel State Information (CSI) imperfection is considered at resource allocation unit. Based on the derived ergodic mutual information, a quantitative characterization of the impact of CSI inaccuracy on the ergodic mutual information is presented. A network with multiple subscriber stations, relay stations and one base station that employ a Selection-Decode-and-Forward (SDF) relaying scheme is considered in the uplink mode. Subscriber stations act as relay stations to assist other transmitting subscriber stations. Numerical evaluations illustrate that considering the CSI imperfection based on priori knowledge of the error statistics brings substantial gain to the network in terms of ergodic mutual information, and takes the MAC layer resource allocation algorithms a step ahead towards practical implementations.**

I. Introduction

Since the first mobile communication system (i.e., Advanced Mobile Phone Services (AMPS)) became commercially available in the mid 1980s, wireless systems have been challenged to support an increasing demand for high data rates and stringent quality of service (QoS) anywhere, any time. Currently, service availability is becoming a must as wireless communications has become an essential part of our daily activities. The absence of wireless coverage could be life threatening for subscribers in emergency situations or patients that are being wirelessly monitored. However, from the service providers' point of view, extending the service coverage in less populated areas is financially expensive as the expected return is less than the infrastructure and operating costs: cabling, land leasing and base stations installation costs. The use of wireless relay technologies reduces the capital cost via wirelessly backhauling the traffic to the wired network.

Because of the limited radio frequency spectrum, the frequency bands to be allocated to the fourth generation (4G) technologies are above 3GHz [1]. Such 4G systems are more vulnerable to no-line-of-sight propagation due to high penetration loss at higher frequencies than at lower frequencies. As a result, signals experience large propagation losses that degrade the transmission performance [2]. Wireless relay technologies take advantage of the broadcasting nature of wireless transmissions, and they re-transmit the received signals to an intended destination. To further exploit the wireless channel capacity, a subscriber station can cooperate with a relay station using a Time Division Duplex (TDD) based Decode-and-Forward (DF) forwarding scheme. In this scheme, the destination receives the direct signal from the transmitter in the first half of the transmission frame and receives the same signal but regenerated by the relay station in the second half [3]. The transmission between the source and relay limits the performance of DF. Alteratively, the relay is selected to cooperate only if the cooperation brings performance improvement in a Selective-DF scheme.

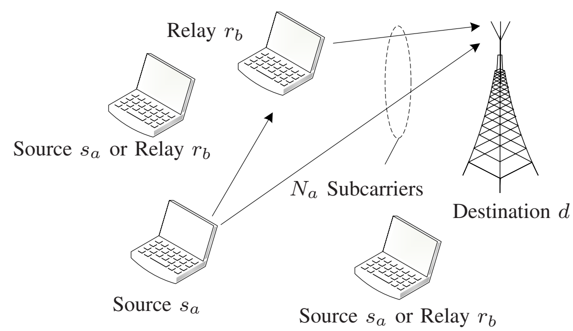

Fig. 1. Illustration of an OFDMA based cooperative relay network.

As more services are being supported (e.g., on demand video streaming and online gaming) in addition to data, voice and video, subscribers strive for high data rates and better QoS support. However, single-carrier signals are limited by the channel coherence bandwidth. As the data rate increases (signal bandwidth increases), the signal becomes subject to inter-symbol-interference (ISI) in frequency selective channels. This challenge can be avoided by transmitting the wideband signal as multiple narrow band signals using the Orthogonal Frequency Division Multiple Access (OFDMA) PHY and MAC technologies [4], [5]. OFDMA is being considered in current broadband standards because of its essential feature of

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exploiting multiuser diversity in a frequency selective channel and eliminating ISI. The outstanding features of OFDMA and relay technologies can be combined in one network architecture called an OFDMA based cooperative relay network. Implementation examples of this architecture are the relay Broadband Wireless Access Networks currently under standardization by the IEEE 802.16j task group [6] and the 3rd Generation Partnership Project (3GPP) [7]. Fig. 1 illustrates an OFDMA cooperative relay network in the uplink mode.

Although extensive research works have been published in the area of relay networks resources allocation, the imperfection of CSI at the resource allocation unit remains a major obstacle in the path to practical implementation. The performance of multi-carrier systems is severely degraded by considering an inaccurate, delayed and probably distorted CSI as perfect and allocating resources based on it [8]–[10]. Thus, the channel knowledge imperfection at the PHY layer propagates to higher layers, resulting in a poor system performance. The aim of this paper is to analyze the ergodic mutual information when the CSI is treated as inaccurate and a priori knowledge of the error statistics is available. Allocating resources based on the derived ergodic mutual information achieves a closer to actual throughput that can be supported than the one that ignores the CSI imperfection. Further, the analysis provides a quantitative insights into the effect of such inaccuracy as various cooperative network parameters vary.

The remainder of the paper is organized as follows. Section II introduces the system model of the OFDMA cooperative relay network under consideration. The SDF ergodic mutual information is derived in section III. The performance improvement achieved by adopting the developed analysis is numerically evaluated in section IV, followed by conclusions in section V.

II. System Model

Consider a single cell scenario with one base station at the center of the cell, multiple relay stations, and multiple subscriber stations. The subscriber stations that are not transmitting can cooperate with other subscriber stations as relay stations. There are A sources forming the set $\mathcal{A} = \{s_1, \cdots, s_a, \cdots, s_A\}$. The available B relays form the set $\mathcal{B} = \{r_1, \cdots, r_b, \cdots, r_B\}$. The destination is symbolized by d. The subscriber stations share a total of N_{sc} subcarriers available to the cell. The set of subcarriers is denoted by $\mathcal{N} = \{1, \cdots, n, \cdots, N_{sc}\}$. In OFDMA networks, a sub-set $\mathcal{N}_a$ [1] of the network subcarriers is exclusively assigned to a source-relay pair, as shown in Fig. 1. We consider a frequency selective fading channel between any pair of communicating stations. However, the subcarriers' narrow bandwidth is assumed to be smaller than the channel coherence bandwidth; therefore, each subcarrier experiences flat fading.

In SDF [3], the transmission frame[2] is divided into two halves. During the first half, the relay r_b and destination d, respectively, receive the following OFDM signals

$$\mathbf{r}^{ab}[j] = \sqrt{\mathbf{P}^a[j]}\mathbf{H}^{ab}[j]\mathbf{s}[j] + \mathbf{z}^{ab}[j] \quad (1)$$

$$\mathbf{r}^{ad}[j] = \sqrt{\mathbf{P}^a[j]}\mathbf{H}^{ad}[j]\mathbf{s}[j] + \mathbf{z}^{ad}[j] \quad (2)$$

where $\sqrt{\mathbf{P}^a}[j]$ is a diagonal $(N_a \times N_a)$ matrix of the vector $\left[\sqrt{p_1^a[j]} \cdots \sqrt{p_n^a[j]} \cdots \sqrt{p_{N_a}^a[j]}\right]$, $p_n^a[j]$ being the power allocated by a MAC resource allocation algorithm to the ath source on the nth subcarrier during the jth slot, $\mathbf{H}^{bd} = diag\{\boldsymbol{\hbar}^{bd}[j]\}$ is the diagonal channel matrix which models the channel between the relay r_b and destination d, $\boldsymbol{\hbar}^{bd}[j] = \left[H_1^{bd}[j] \cdots H_n^{bd}[j] \cdots H_{N_a}^{bd}[j]\right]$ with $H_n^{bd}[j]$ being the nth subcarrier gain of the r_b to d channel during the jth slot[3]. $\mathbf{s}[j]$ denotes the data source symbols. The vectors $\mathbf{z}^{ab}$ and $\mathbf{z}^{ad}$, respectively, represent the additive noise at the source-to-relay and source-to-destination channels which are modeled as circularly symmetric complex Gaussian $\mathbf{z}^{ab} \sim \mathcal{CN}(\mathbf{0}, (\sigma_z^{ab})^2\mathbf{I})$ and $\mathbf{z}^{ad} \sim \mathcal{CN}(\mathbf{0}, (\sigma_z^{ad})^2\mathbf{I})$. In the second half, if the source-to-relay ergodic mutual information is greater than a threshold R, the relay cooperates with the source, and the destination receives the following OFDM signal

$$\mathbf{r}^{bd}[j + \frac{T_f}{2}] = \sqrt{\mathbf{P}^b}[j + \frac{T_f}{2}](\sqrt{\mathbf{P}^a}[j + \frac{T_f}{2}])^{-1} \times\mathbf{H}^{bd}[j + \frac{T_f}{2}]\hat{\mathbf{r}}^{ab}[j] + \mathbf{z}^{bd}[j + \frac{T_f}{2}], \quad (3)$$

where T_f is the frame length and $\sqrt{\mathbf{P}^b}[j]$ is a diagonal matrix of the vector $\left[\sqrt{p_1^b[j]} \cdots \sqrt{p_n^b[j]} \cdots \sqrt{p_{N_b}^b[j]}\right]$, with $p_n^b[j]$ being the power allocated to the bth relay on the nth subcarrier during the jth slot. $\hat{\mathbf{r}}^{ab}[j]$ denotes the re-encoded signal by r_b. The additive noise vector at the relay-to-destination channel $\mathbf{z}^{bd}[j + \frac{T_f}{2}]$ is modeled as $\mathbf{z}^{bd} \sim \mathcal{CN}(\mathbf{0}, (\sigma_z^{bd})^2\mathbf{I})$. The destination d combines the received signals ($\mathbf{r}^{ad}[j]$ and $\mathbf{r}^{bd}[j + \frac{T_f}{2}]$) by employing one of the literature available diversity combining schemes [3]. Conversely, if the mutual information of the s_a to r_b link is less than R, s_a continues transmitting to d without the cooperation of r_b.

The CSI is updated every OFDMA frame. As mentioned previously, the frame is divided into two sub-frames. At the beginning of the first half of the frame, a sequence of OFDM symbols is transmitted by the source s_a to the relay r_b and destination d for channel estimation. In the second sub-frame, the relay r_b transmits another set of training symbols for the destination d to estimate the channel. In addition, it forwards its estimate $\hat{\mathbf{H}}^{ab}$ of $\mathbf{H}^{ab}$ to the destination d which estimates $\mathbf{H}^{bd}$ and $\mathbf{H}^{ad}$ to obtain $\hat{\mathbf{H}}^{bd}$ and $\hat{\mathbf{H}}^{ad}$. The slot index (i.e., $[j]$) is dropped for simpler notation. Fig. 2 shows the above mentioned modeling parameters on a model network of a source-relay pair and a destination.

Note that the channel matrices are diagonals of the subcarriers channel gain vectors, namely $\boldsymbol{\hbar}^{ab}$, $\boldsymbol{\hbar}^{ad}$ and $\boldsymbol{\hbar}^{bd}$. Let $\hat{\boldsymbol{\hbar}}^{ab}$, $\hat{\boldsymbol{\hbar}}^{ad}$ and $\hat{\boldsymbol{\hbar}}^{bd}$ be their estimates available at the receiver.

[1]The cardinality of the sub-set $\mathcal{N}_a$ is denoted by N_a.

[2]An OFDMA frame consists of multiple time slots. A OFDMA symbol is transmitted on all assigned frequencies during the same slot [5].

[3]The superscript $\cdot^{ab}$, $\cdot^{bd}$ and $\cdot^{ad}$, respectively, denote the link between a source s_a and a relay r_b, a relay r_b and a source s_a, and a source s_a and the destination d.

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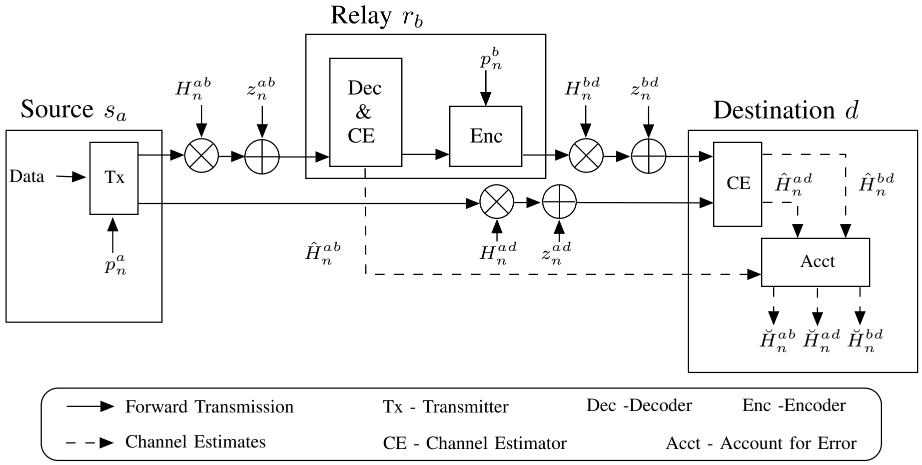

Fig. 2. Illustrative SDF network showing the actual, estimated and imperfect CSI on the three links.

Before the next frame estimates arrive, they are treated as deterministic [11] and their delay and estimation error are modeled by $\tilde{\boldsymbol{\hbar}}^{ab}$, $\tilde{\boldsymbol{\hbar}}^{ad}$ and $\tilde{\boldsymbol{\hbar}}^{bd}$ [12]. Hence, given the channel estimates $\hat{\boldsymbol{\hbar}}^{ab}$, $\hat{\boldsymbol{\hbar}}^{ad}$ and $\hat{\boldsymbol{\hbar}}^{bd}$, the imperfect CSI for the three links (i.e., s_a to r_b, s_a to d and r_b to d) are modeled, respectively, as follows:

$$\breve{\boldsymbol{\hbar}}^{ab} = \hat{\boldsymbol{\hbar}}^{ab} + \tilde{\boldsymbol{\hbar}}^{ab}; \tag{4}$$

$$\breve{\boldsymbol{\hbar}}^{ad} = \hat{\boldsymbol{\hbar}}^{ad} + \tilde{\boldsymbol{\hbar}}^{ad}; \tag{5}$$

$$\breve{\boldsymbol{\hbar}}^{bd} = \hat{\boldsymbol{\hbar}}^{bd} + \tilde{\boldsymbol{\hbar}}^{bd}. \tag{6}$$

$\breve{\boldsymbol{\hbar}}^{ab}$, $\breve{\boldsymbol{\hbar}}^{ad}$ and $\breve{\boldsymbol{\hbar}}^{bd}$ are, respectively, assumed to be $\sim \mathcal{CN}(\hat{\boldsymbol{\hbar}}^{ab}, \boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{ab}})$, $\sim \mathcal{CN}(\hat{\boldsymbol{\hbar}}^{ad}, \boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{ad}})$ and $\sim \mathcal{CN}(\hat{\boldsymbol{\hbar}}^{bd}, \boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{bd}})$ where $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{ab}}$, $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{ad}}$ and $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{bd}}$ are the error covariance matrices [12], [13]. We assume that the estimation error on different subcarriers are independent; hence, the covariance matrix is a scalar multiple of the identity matrix. Therefore, $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{ab}} = (\tilde{\sigma}_\epsilon^{ab})^2\mathbf{I}$, $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{ad}} = (\tilde{\sigma}_\epsilon^{ad})^2\mathbf{I}$, and $\boldsymbol{\Sigma}_{\tilde{\boldsymbol{\hbar}}^{bd}} = (\tilde{\sigma}_\epsilon^{bd})^2\mathbf{I}$ where $(\tilde{\sigma}_\epsilon^{ab})^2$, $(\tilde{\sigma}_\epsilon^{ad})^2$, and $(\tilde{\sigma}_\epsilon^{bd})^2$ are the delay and estimation error variances. Hence, the nth subcarrier[4] imperfect CSI ($[\breve{\boldsymbol{\hbar}}^{ab}]_n = \breve{H}_n^{ab}$, $[\breve{\boldsymbol{\hbar}}^{ad}]_n = \breve{H}_n^{ad}$, and $[\breve{\boldsymbol{\hbar}}^{bd}]_n = \breve{H}_n^{bd}$) are, respectively, modeled as $\sim \mathcal{CN}(\hat{H}_n^{ab}, (\tilde{\sigma}_\epsilon^{ab})^2)$, $\sim \mathcal{CN}(\hat{H}_n^{ad}, (\tilde{\sigma}_\epsilon^{ad})^2)$ and $\sim \mathcal{CN}(\hat{H}_n^{bd}, (\tilde{\sigma}_\epsilon^{bd})^2)$. Therefore, their squares follow non-central chi-square probability density functions (PDF) given by [14] as follows:

$$f_X(x) = \frac{1}{(\tilde{\sigma}_\epsilon^{ad})^2} e^{-\frac{(|\hat{H}_n^{ad}|^2 + x)}{(\tilde{\sigma}_\epsilon^{ad})^2}} \Im_0\left(2\sqrt{\frac{|\hat{H}_n^{ad}|^2 x}{(\tilde{\sigma}_\epsilon^{ad})^4}}\right), \tag{7}$$

$$f_Y(y) = \frac{1}{(\tilde{\sigma}_\epsilon^{ab})^2} e^{-\frac{(|\hat{H}_n^{ab}|^2 + y)}{(\tilde{\sigma}_\epsilon^{ab})^2}} \Im_0\left(2\sqrt{\frac{|\hat{H}_n^{ab}|^2 y}{(\tilde{\sigma}_\epsilon^{ab})^4}}\right), \tag{8}$$

[4] $[\boldsymbol{x}]_n$ denotes the nth element of vector $\boldsymbol{x}$.

$$f_Z(z) = \frac{1}{(\tilde{\sigma}_\epsilon^{bd})^2} e^{-\frac{(|\hat{H}_n^{bd}|^2 + z)}{(\tilde{\sigma}_\epsilon^{bd})^2}} \Im_0\left(2\sqrt{\frac{|\hat{H}_n^{bd}|^2 z}{(\tilde{\sigma}_\epsilon^{bd})^4}}\right), \tag{9}$$

where $\Im_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. The random variables $|\breve{H}_n^{ad}|^2$, $|\breve{H}_n^{ab}|^2$ and $|\breve{H}_n^{bd}|^2$ are, respectively, denoted by X, Y and Z for simpler notation.

III. SDF Ergodic Mutual Information

The SDF mutual information for a deterministic CSI is given by equation (10) [3]. When the imperfection of CSI is considered, the channel gains are random variables and the ergodic mutual information becomes a function of them; thus, (10) becomes

$$E\left[I_{n,a,b}^{SDF}\right] = \begin{cases} E\left[I_n^{ad}|I_n^{ab}\right], & Pr\left\{I_n^{ab} \le R\right\} \quad \text{(11a)} \\ E\left[I_n^{abd}|I_n^{ab}\right], & Pr\left\{I_n^{ab} > R\right\}, \quad \text{(11b)} \end{cases}$$

where $E\left[I_n^{ad}|I_n^{ab}\right]$ is the ergodic mutual information of the direct transmission and $E\left[I_n^{abd}|I_n^{ab}\right]$ is the cooperation ergodic mutual information, $Pr\left\{I_n^{ab} \le R\right\}$ is the probability that the information on the source-to-relay channel is less than or equal to a threshold R. Given the PDF of $|\breve{H}_n^{ad}|^2$, $|\breve{H}_n^{ab}|^2$ and $|\breve{H}_n^{bd}|^2$, we derive the ergodic mutual information of SDF, $E\left[I_{n,a,b}^{SDF}\right]$. In the following, we focus on the mutual information at the subcarrier level; thus, the subcarrier index n is removed.

A. Direct and Cooperative Transmission Probabilities: $Pr\left\{I^{ab} \le R\right\}$ and $Pr\left\{I^{ab} > R\right\}$

The random variable mutual information between s_a and r_b that is function of the imperfect CSI random variable Y is given by [12]

$$I^{ab} = \frac{1}{2}\log_e\left(1 + \frac{2p^a Y}{(\sigma_z^{ad})^2}\right), \tag{12}$$

then,

$$Pr\left\{I^{ab} \le R\right\} = \int_0^R f_{I^{ab}}(i)di, \tag{13}$$

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$$\overline{I}_{n,a,b}^{SDF} = \begin{cases} \frac{1}{2}\log\left(1+\frac{2p_n^a|\hat{H}_n^{ad}|^2}{(\sigma_z^{ad})^2}\right), & \frac{1}{2}\log\left(1+\frac{2p_n^a|\hat{H}_n^{ab}|^2}{(\sigma_z^{ab})^2}\right)\leq R \quad (10a)\\ \frac{1}{2}\log\left(1+\frac{2p_n^a|\hat{H}_n^{ad}|^2}{(\sigma_z^{ad})^2}+\frac{2p_n^b|\hat{H}_n^{bd}|^2}{(\sigma_z^{bd})^2}\right), & \frac{1}{2}\log\left(1+\frac{2p_n^a|\hat{H}_n^{ab}|^2}{(\sigma_z^{ab})^2}\right)> R \quad (10b)\end{cases}$$

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where $f_{I^{ab}}(i)$ is the PDF of I^{ab}, i.e.,

$$f_{I^{ab}}(i) = 2\zeta e^{2i-\eta-\zeta(e^{2i}-1)}\Im_0\left(2\sqrt{\eta\zeta(e^{2i}-1)}\right), \quad (14)$$

for $\eta = \frac{|\hat{H}^{ab}|^2}{(\tilde{\sigma}_\epsilon^{ab})^2}$ and $\zeta = \frac{(\sigma_z^{ab})^2}{2p^a(\tilde{\sigma}_\epsilon^{ab})^2}$. By the transformation $i' = (e^{2i}-1)$, inserting the series representation of $\Im_0(\cdot)$ ([15], 8.447(1)) into (14), and (14) into (13) gives

$$Pr\left\{I^{ab}\leq R\right\} = \int_0^{e^{2R}-1} e^{-\eta-\zeta i'}\sum_{k=0}^{\infty}\frac{(\eta\zeta i')^k}{(k!)^2}di'. \quad (15)$$

Now, integrating by parts and re-arranging the absolutely convergent series [16], the above integral in (15) evaluates to

$$= e^{-\eta}\left[\sum_{m=0}^{\infty} -e^{-\zeta(e^{2R}-1)}\sum_{k'=0}^{m}\frac{\eta^m(\zeta(e^{2R}-1))^{k'}}{m!k'!}+\frac{\eta^m}{m!}\right]. \quad (16)$$

By substituting back the values of η and ζ, (16) becomes

$$Pr\left\{I^{ab}\leq R\right\} = e^{-\frac{|\hat{H}^{ab}|^2}{(\tilde{\sigma}_\epsilon^{ab})^2}}\left[\sum_{m=0}^{\infty}\left(-e^{-\left(\frac{(\sigma_z^{ab})^2}{2p^a(\tilde{\sigma}_\epsilon^{ab})^2}\right)(e^{2R}-1)}\times\sum_{k'=0}^{m}\frac{\left(\frac{|\hat{H}^{ab}|^2}{(\tilde{\sigma}_\epsilon^{ab})^2}\right)^m\left(\left(\frac{(\sigma_z^{ab})^2}{2p^a(\tilde{\sigma}_\epsilon^{ab})^2}\right)(e^{2R}-1)\right)^{k'}}{m!k'!}\right)+\frac{\left(\frac{|\hat{H}^{ab}|^2}{(\tilde{\sigma}_\epsilon^{ab})^2}\right)^m}{m!}\right]. \quad \square \quad (17)$$

Hence, $Pr\left\{I^{ab}>R\right\}$ can be simply found from (17) by

$$Pr\left\{I^{ab}>R\right\} = 1 - Pr\left\{I^{ab}\leq R\right\}. \quad (18)$$

B. Conditional Direct Transmission Ergodic Mutual Information $E\left[I^{ad}|I^{ab}\right]$

If the mutual information of the s_a to r_b channel is less than the threshold R, the relay does not cooperate and the source transmits directly to the destination d. The random variable I^{ad} is a function of the imperfect CSI random variable X (i.e., $|\breve{H}_n^{ad}|^2$) defined as follows

$$I^{ad} = \frac{1}{2}\log_e\left(1+\frac{2p^aX}{(\sigma_z^{ad})^2}\right). \quad (19)$$

The random variable $Q = \frac{2p^aX}{(\sigma_z^{ad})^2}$ PDF is

$$f_Q(q) = \frac{1}{\Omega_Q^2}e^{-\frac{\alpha_Q^2+q}{\Omega_Q^2}}\Im_0\left(2\sqrt{\frac{\alpha_Q^2 q}{\Omega_Q^4}}\right), \quad (20)$$

where $\frac{1}{\Omega_Q^2} = \frac{(\sigma_z^{ad})^2}{2p^a(\tilde{\sigma}_\epsilon^{ad})^2}$ and $\alpha_Q^2 = \frac{2p^a|\hat{H}^{ad}|^2}{(\sigma_z^{ad})^2}$. By substituting the series representation of $\Im_0(\cdot)$ ([15], 8.447(1)) in (20), we obtain

$$f_Q(q) = \frac{1}{\Omega_Q^2}e^{-\frac{\alpha_Q^2+q}{\Omega_Q^2}}\sum_{t=0}^{\infty}\frac{\alpha_Q^{2t}q^t}{\Omega_Q^{4t}(t!)^2}. \quad (21)$$

Given the PDF $f_Q(q)$, the ergodic mutual information for direct communication can be written as

$$E\left[I^{ad}|I^{ab}\right] = \int_0^{\infty}\frac{1}{2}\log_e(1+q)f_Q(q)dq \quad (22)$$

$$= \frac{e^{-\frac{\alpha_Q^2}{\Omega_Q^2}}}{2\Omega_Q^2}\sum_{t=0}^{\infty}\frac{\alpha_Q^{2t}}{\Omega_Q^{4t}(t!)^2}\int_0^{\infty}\log_e(1+q)e^{-\frac{q}{\Omega_Q^2}}q^t dq. \quad (23)$$

By ([15], 4.222(8)), we obtain

$$E\left[I^{ad}|I^{ab}\right] = \frac{e^{-\frac{\alpha_Q^2}{\Omega_Q^2}}}{2\Omega_Q^2}\sum_{t=0}^{\infty}\frac{\alpha_Q^{2t}\Omega_Q^{2(t+1)}}{\Omega_Q^{4t}(t!)^2}\sum_{m=0}^{t}\frac{t!}{(t-m)!}\times\left[\frac{(-1)^{t-m-1}}{\Omega_Q^{2(t-m)}}e^{\frac{1}{\Omega_Q^2}}Ei\left(\frac{-1}{\Omega_Q^2}\right)+\sum_{j=1}^{t-m}\frac{(j-1)!}{-\Omega_Q^{2(t-m-j)}}\right], \quad (24)$$

where $Ei(\cdot)$ is the exponential integral function. $\square$

C. Conditional Cooperative Transmission Ergodic Mutual Information $E\left[I^{abd}|I^{ab}\right]$

Define the random variables $W = \frac{2p^bZ}{(\sigma_z^{bd})^2}$, $S = Q + W$ and

$$I^{abd} = \frac{1}{2}\log_e\left(1+\frac{2p^aX}{(\sigma_z^{ad})^2}+\frac{2p^bZ}{(\sigma_z^{bd})^2}\right), \quad (25)$$

where the PDF of Z is given by (9). The PDF of Q was found in (20) and the PDF of W is given by

$$f_W(w) = \frac{(\sigma_z^{bd})^2 e^{-\frac{|\hat{H}^{bd}|^2+\frac{(\sigma_z^{bd})^2 w}{2p^b}}{(\tilde{\sigma}_\epsilon^{bd})^2}}}{2p^b(\tilde{\sigma}_\epsilon^{bd})^2}\Im_0\left(2\sqrt{\frac{|\hat{H}^{bd}|^2(\sigma_z^{bd})^2 w}{2p^b(\tilde{\sigma}_\epsilon^{bd})^4}}\right). \quad (26)$$

Both $f_Q(q)$ and $f_W(w)$ can be mapped to the type-one Bessel function PDF [17], respectively, for $\theta_Q = \frac{(\sigma_z^{ad})^2}{2p^a(\tilde{\sigma}_\epsilon^{ad})^2}$, $\beta_Q^2 = \frac{4|\hat{H}^{ad}|^2(\sigma_z^{ad})^2}{2p^a(\tilde{\sigma}_\epsilon^{bd})^4}$ and $\lambda_Q = 1$, and $\theta_W = \frac{(\sigma_z^{bd})^2}{2p^b(\tilde{\sigma}_\epsilon^{bd})^2}$, $\beta_W^2 = \frac{4|\hat{H}^{bd}|^2(\sigma_z^{bd})^2}{2p^b(\tilde{\sigma}_\epsilon^{bd})^2}$ and $\lambda_W = 1$.

Since the destination d is common to both (r_b to d) and (s_a to d) transmissions, the estimation and delay error, and

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the additive noise variance are equal (i.e., $(\sigma_z^{ad})^2 = (\sigma_z^{bd})^2$ and $(\tilde{\sigma}_\epsilon^{ad})^2 = (\tilde{\sigma}_\epsilon^{bd})^2$). In cooperative networks with the relay r_b being one of the subscriber stations assisting another subscriber station, the source and the relay are assumed to be transmitting at equal power, i.e., $p^a = p^b$. Thus, $\theta_Q = \theta_W$. By the reproductive property of Bessel function random variables [18], the PDF of S is given by

$$f_S(s) = \frac{2\theta_S^2}{\beta_S} e^{\frac{-\beta_S^2}{4\theta_S}} s^{\frac{1}{2}} e^{-\theta_S s} \sum_{k=0}^{\infty} \frac{\left(\sqrt{\beta_S^2 s/4}\right)^{2k+1}}{k!(k+1)!}, \qquad (27)$$

where $\theta_S = \theta_Q = \theta_W$ (i.e., common variance), $\beta_S^2 = \beta_Q^2 + \beta_W^2$. Given the PDF of the sum random variable S, the ergodic mutual information is written as

$$E\left[I^{abd}|I^{ab}\right] = \int_0^\infty \frac{1}{2}\log_e(1+s) f_S(s) ds. \qquad (28)$$

Substituting (27) in (28) and using ([15], 8.444(8)), equation (28) becomes

$$E\left[I^{abd}|I^{ab}\right] = \frac{e^{-\frac{\beta_S^2}{4\theta_S}}}{\beta_S} \sum_{k=0}^{\infty} \frac{\theta_S^{-k}\left(\frac{\beta_S}{2}\right)^{2k+1}}{k!} \sum_{m=0}^{k+1} \frac{1}{(k-m+1)!} \times \left[\frac{(-1)^{k-m}}{\left(\frac{1}{\theta_S}\right)^{k-m+1}} e^{\theta_S} Ei(-\theta_S) + \sum_{l=1}^{k-m+1} \frac{(l-1)!}{\left(\frac{-1}{\theta_S}\right)^{k-m-l+1}} \right] . \Box \qquad (29)$$

The SDF ergodic mutual information on each subcarrier is obtained by substituting the direct and cooperative transmission probabilities (17) and (18), the direct transmission ergodic mutual information (24), and the cooperative transmission ergodic mutual information (29) in

$$E\left[I_{n,a,b}^{SDF}\right] = E\left[I^{ad}|I^{ab}\right] Pr\left\{I^{ab} \le R\right\} + E\left[I^{abd}|I^{ab}\right] Pr\left\{I^{ab} > R\right\}. \qquad (30)$$

Whereas equation (30) is specific to each subcarrier assigned to a subscriber-relay pair, it can be generalized to capture the network ergodic mutual information as follows

$$E\left[I_{\text{Net}}^{SDF}\right] = \sum_{n=1}^{N_{sc}} \sum_{a=1}^{A} \sum_{b=1}^{B} E\left[I_{n,a,b}^{SDF}\right]. \qquad (31)$$

IV. Numerical Evaluations

Consider a SDF network with a subscriber station, a relay station and a destination. The numerical analysis focuses on comparing two scenarios. The *first scenario* represents networks that allocate resources based on the assumption that the received CSI for each subcarrier is perfect, i.e., $|\hat{H}_n^{ab}|^2 = |H_n^{ab}|^2$, $|\hat{H}_n^{ad}|^2 = |H_n^{ad}|^2$ and $|\hat{H}_n^{bd}|^2 = |H_n^{bd}|^2$ (Fig. 2). The resources allocation unit allocates power and subcarriers, and pairs cooperating stations which are expected to achieve the per-subcarrier mutual information, $\overline{I}_{n,a,b}^{SDF}$, given in equation (10). The *second scenario* represents more practical networks, in which the estimates of the channels are considered inaccurate, and the inaccuracy is based on a prior knowledge of the errors statistics as in (4), (5) and (6). Based on $|\breve{H}_n^{ab}|^2$, $|\breve{H}_n^{ad}|^2$ and $|\breve{H}_n^{bd}|^2$ (Fig. 2), the source achieves the ergodic mutual information $E\left[I_{n,a,b}^{SDF}\right]$ derived in equation (30) on each subcarrier assigned to it.

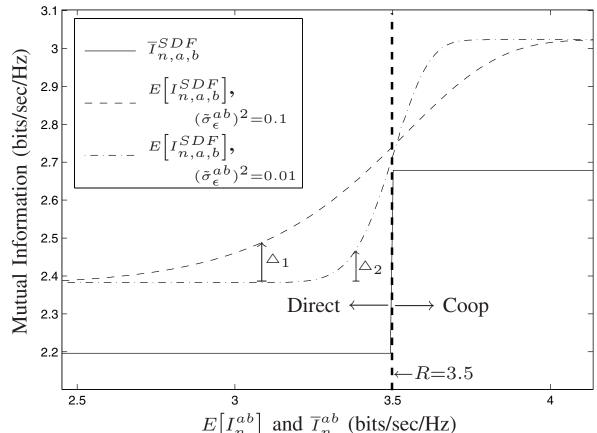

Fig. 3. Change in mutual information as the direct (i.e., s_a to r_b) mutual information increases.

Increasing the signal-to-noise ratio (SNR) on the s_a to r_b channel from 3dB to 23dB, increases the source-to-relay mutual information, represented by the horizontal axis in Fig. 3. Thus, s_a switches from direct transmission mode to the cooperative mode as shown in Fig. 3 for $(\tilde{\sigma}_\epsilon^{ad})^2 = (\tilde{\sigma}_\epsilon^{bd})^2 = 0.1$ and equal SNR of 10dB on the s_a to d and r_b to d channels. The cooperation decision is based on the deterministic (i.e., $\overline{I}_n^{ab}$) and ergodic (i.e., $E\left[I_n^{ab}\right]$) mutual information on the s_a to r_b channel in the first and second scenario, respectively. Thus, considering an inaccurate channel estimate as a perfect one leads to the wrong decision as evident from Fig. 3. Because of the estimation and delay errors, the first scenario subscriber stations do not cooperate although the actual channel condition is at cooperation permissible level. On the other hand, the second scenario networks consider the error and takes advantage of the channel gain increase which translates to higher mutual information as indicated by $\triangle_1$ and $\triangle_2$ on Fig. 3. It is obvious that higher mutual information can be achieved by the second scenario networks in either cooperative or direct transmission modes.

Fig. 4 shows the percentage increase in the mutual information achieved, $\frac{E\left[I_{n,a,b}^{SDF}\right] - \overline{I}_{n,a,b}^{SDF}}{\overline{I}_{n,a,b}^{SDF}}$ %, when the CSI uncertainty is considered at the receiver as the SNR on the three channels varies and the threshold R increases for equal error variances, $(\tilde{\sigma}_\epsilon^{ab})^2 = (\tilde{\sigma}_\epsilon^{ad})^2 = (\tilde{\sigma}_\epsilon^{bd})^2 = 0.01$. Although the estimation and delay error is more significant at high SNR compared to low SNR [8], the percentage increase in mutual information is more pronounced at low SNR than at high SNR. This observation is explained as follows: the large gain achieved by considering the second scenario relative to the mutual information of the first scenario at high SNR is less than the small gain achieved by considering the second scenario relative to the mutual information of the first scenario at low SNR. The sharp increase seen in Fig. 4 is due to the difference

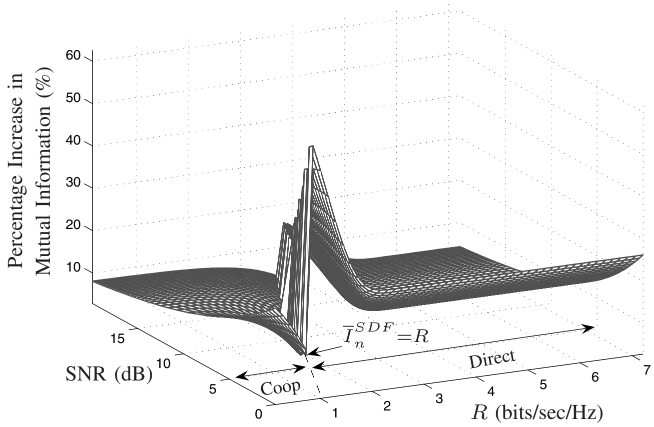

Fig. 4. Percentage increase in mutual information when CSI imperfection is taken into account in both cooperative (Coop) and direct (Direct) transmission modes.

in transmission modes for both scenarios. In other words, the first scenario r_b switches to direct transmission mode based on the poor estimate $|\hat{H}_n^{ab}|^2$, while the second scenario r_b remains in the cooperation mode based on a better estimate, $|\breve{H}_n^{ab}|^2$, than $|\hat{H}_n^{ab}|^2$.

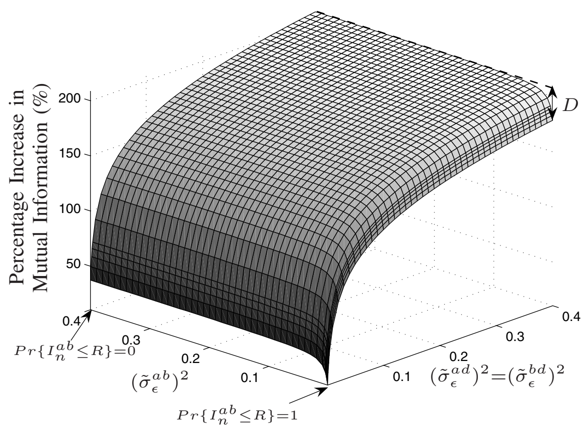

Fig. 5. Effect of CSI inaccuracy on the ergodic mutual information.

To investigate the impact of the three links CSI imperfection on the ergodic mutual information, Fig. 5 shows the ergodic mutual information gain achieved in the second scenario relative to first scenario as the three errors increase for a constant SNR of 10dB on the three links. Since estimation and delay errors at the relay affect the transmission mode decision (i.e., cooperative vs. direct), as $(\tilde{\sigma}_\epsilon^{ab})^2$ increases, the cooperative transmission probability increases, resulting in a gain denoted by D over the direct transmission. The increase in $(\tilde{\sigma}_\epsilon^{ad})^2$ and $(\tilde{\sigma}_\epsilon^{bd})^2$ affects both cooperative and direct transmissions ergodic mutual information; hence, the second scenario achieves larger gain over the first scenario as the estimation and delay errors increase.

V. CONCLUSIONS

We have analyzed the ergodic mutual information of OFDMA based cooperative relay networks that employ the SDF relaying scheme. Further, considering the CSI imperfection at the resource allocation unit, has been shown to increase the ergodic mutual information over a large range of SNR. However, the relative increase is more tangible at low SNR than that at high SNR. Therefore, until designed resource allocation algorithms consider channel estimates and delay errors, they remain far from being practically implementable and from achieving their claimed performance in real networks. Our ongoing work is to design a CSI imperfection resilient resource allocation algorithm for SDF networks.

REFERENCES

[1] T. Otsu, I. Okajima, N. Umeda, and Y. Yamao, "Network architecture for mobile communications systems beyond IMT-2000," *IEEE Personal Communications*, vol. 8, no. 5, pp. 31–37, Oct 2001.

[2] M. K. Awad, K. T. Wong, and Z. Li, "An integrative overview of the open literatures empirical data on the indoor radiowave channels temporal properties," *IEEE Transactions on Antenna and Proagation*, vol. 56, no. 5, pp. 1451 – 1468, 2008.

[3] J. Laneman, D. Tse, and G. Wornell, "Cooperative diversity in wireless networks: Efficient protocols and outage behavior," *IEEE Transactions on Information Theory*, vol. 50, no. 12, pp. 3062–3080, 2004.

[4] M. K. Awad and X. S. Shen, "OFDMA based two-hop cooperative relay network resources allocation," in *Proc. IEEE International Conference on Communications (ICC'08)*, Beijing, China, May 2008.

[5] M. Mehrjoo, M. K. Awad, and X. S. Shen, *WiMAX Network Planning and Optimization*. CRC Press - Taylor & Francis Group, to Appear, ch. Optimum Power and Subcarrier Allocation in OFDM WiMAX.

[6] "Draft standard for local and metropolitan area networks–part 16: Air interface for fixed and mobile broadband wireless access systems–multihop relay specification," *IEEE Unapproved Draft Std P802.16j/D1*, 2007.

[7] "3rd generation partnership project (3GPP) specifications home page," [Online]. Available: http://www.3gpp.org/specs/specs.htm, (2008).

[8] A. Leke and J. Cioffi, "Impact of imperfect channel knowledge on the performance of multicarrier systems," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM'98)*, Sydney, NSW, Australia, Aug. 1998, pp. 951–955.

[9] H. Lichte, S. Valentin, F. Eitzen, M. Stege, C. Unger, and H. Karl, "Integrating multiuser dynamic OFDMA into IEEE 802.11 a and prototyping it on a real-time software-defined radio testbed," in *Proc. Intl. Conf. on Testbeds and Research Infrastructures for the Development of Networks and Communities (TridentCom'07)*, Orlando, Florida, May 2007, pp. 1–9.

[10] J. Gross, H. Karl, and A. Wolisz, "On the effect of inband signaling and realistic channel knowledge on dynamic OFDM-FDMA systems," in *Proc. of European Wireless*, Barcelona, Spain, Feb. 2004.

[11] S. Zhou and G. Giannakis, "Optimal transmitter eigen-beamforming and space-time block coding based on channel mean feedback," *IEEE Transactions on Signal Processing*, vol. 50, no. 10, pp. 2599–2613, 2002.

[12] Y. Yao and G. Giannakis, "Rate-maximizing power allocation in OFDM based on partial channel knowledge," *IEEE Transactions on Wireless Communications*, vol. 4, no. 3, pp. 1073–1083, 2005.

[13] I. C. Wong and B. L. Evans, "OFDMA downlink resource allocation for ergodic capacity maximization with imperfect channel knowledge," in *Proc. IEEE Global Telecommunications Conference (GLOBECOM' 07)*, Washington, DC, Feb 2007.

[14] J. Proakis *et al.*, *Digital Communication*. Osborne-McGraw-Hill, 2001.

[15] I. Gradshteĭn, I. Ryzhik, D. Zwillinger, and A. Jeffrey, *Table of Integrals, Series, and Products*. Academic Press, 2007.

[16] G. Arfken, *Mathematical Methods for Physicists*. Academic Press, 1966.

[17] M. Springer, *The Algebra of Random Variables*. J. Wiley & Sons, 1979.

[18] R. G. Laha, "On some properties of the Bessel function distributions," *Bulletin of the Calcutta Mathematical Society*, vol. 46, pp. 59 – 71, 1954.