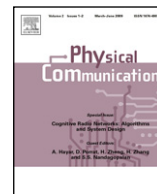




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Ergodic mutual information of OFDMA-based Selection-Decode-and-Forward cooperative relay networks with imperfect CSI[☆]

Mohamad Khattar Awad^a, Xuemin (Sherman) Shen^{a,*}, Bashar Zogheib^b

^a Department of Electrical & Computer Engineering, University of Waterloo, Waterloo, Ontario, N2L 3G1, Canada

^b Division of Math, Science, and Technology, Nova Southeastern University, Fort Lauderdale, FL, 33314, United States

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ABSTRACT

This paper presents a novel approach to investigate ergodic mutual information of OFDMA Selection-Decode-and-Forward (SDF) cooperative relay networks with imperfect channel state information (CSI). Relay stations are either dedicated or non-dedicated (i.e., subscriber stations assisting other subscriber stations). The CSI imperfection is modeled as an additive random variable with known statistics. Numerical evaluations and simulations demonstrate that by considering the CSI imperfection based on a priori knowledge of the estimation error statistics, a substantial gain can be achieved in terms of ergodic mutual information which makes channel adaptive schemes closer to practical implementations.

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1. Introduction

Since the first mobile communication system (i.e., Advanced Mobile Phone Services (AMPS)) became commercially available in the mid-1980s, wireless systems have been challenged to support an increasing demand for high data rates and stringent quality of service (QoS) anywhere, at any time. The absence of wireless coverage could be life threatening for service subscribers in emergency situations or patients who are being wirelessly monitored. However, from the service providers' point of view, extending the

service coverage in less populated areas is financially expensive as the expected revenue is less than the infrastructure and operating costs: cabling, land leasing and base station installation costs. The use of wireless relay technologies reduces these costs via wirelessly backhauling the traffic to the wired network.

Because of the limited radio frequency spectrum, the frequency bands to be allocated to fourth-generation (4G) technologies are above 3 GHz [1]. Such 4G systems are more vulnerable to no-line-of-sight propagation due to high penetration loss at high frequencies. As a result, signals experience large propagation losses that degrade transmission performance [2]. Wireless relay technologies take advantage of the broadcasting nature of wireless transmissions to retransmit received signals to an intended destination. To further exploit wireless channel capacity, a subscriber station can cooperate with a relay station through a Time Division Duplex (TDD) based Decode-and-Forward (DF) relaying scheme. In this way, the destination receives the direct signal from the transmitter in the first half of the transmission frame and receives the same signal but regenerated by the relay station in the second half [3]. The transmission between the source and relay limits the performance of DF. Thus, DF can be further improved

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* Corresponding author. Tel.: +1 519 888 4567x32691; fax: +1 519 746 3077.

E-mail addresses: Mohamad@ieee.org (M.K. Awad), xshen@bbcr.uwaterloo.ca (X. Shen), zogheib@nova.edu (B. Zogheib).

URLs: <http://www.MohamadAwad.com> (M.K. Awad), <http://bbcr.uwaterloo.ca/~xshen> (X. Shen).

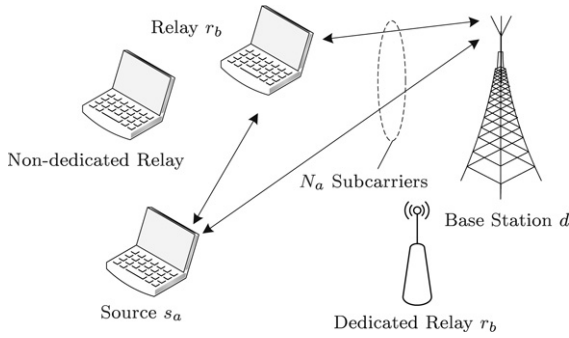


Fig. 1. Illustration of an OFDMA-based cooperative relay network.

by selecting the relay to cooperate only if cooperation brings performance improvement in a Selection-Decode-and-Forward scheme. The relay technology properties of reducing the propagation loss and the deployment cost in addition to exploiting spatial diversity have been receiving a lot of attention by various standards working groups.

As more services are being supported (e.g., on-demand video streaming and online gaming) in addition to data, voice and video, subscribers strive for high data rates and better QoS support; however, single-carrier signals are limited by the channel coherence bandwidth. As the data rate increases (signal bandwidth increases), the signal becomes subject to intersymbol interference (ISI) in frequency selective channels. This challenge can be overcome if the wide-band signal is transmitted as multiple narrow band signals using the Orthogonal Frequency Division Multiple Access (OFDMA) PHY and MAC technologies [4,5]. OFDMA is being considered in current broadband standards because of its essential features of exploiting multiuser diversity in a frequency selective channel and eliminating ISI. The outstanding features of OFDMA and relay technologies can be combined into one network architecture called an OFDMA-based cooperative relay network. Implementation examples of this architecture are the relay Broadband Wireless Access Networks currently under standardization by the IEEE 802.16j task group [6] and the 3rd Generation Partnership Project (3GPP) [7]. Fig. 1 illustrates a single cell scenario of OFDMA cooperative relay networks.

Although extensive research has been published on relay networks, the inaccuracy of CSI remains a major obstacle for their practical implementation. The performance of multicarrier systems is severely degraded by considering an inaccurate and delayed CSI as a perfect one [8–10], where the channel estimation error at the PHY layer propagates to higher layers, resulting in poor system performance. This practical implementation issue has motivated researchers working on different aspects of wireless networks design to consider estimation error in their design. For instance, a relay-precoder and decoder are jointly optimized for cooperative networks in [11]. In [12], a power loading scheme for data and pilots is proposed for OFDMA networks. Resource allocation (i.e., power, rate and beamforming) schemes are proposed for OFDMA (point-to-multipoint) PMP networks [13–15] and for cooperative networks [16]. The design of a space-time constellation is considered in [17]. An extensive survey on the field of

limited feedback across different systems and standards can be found in [18]. In all mentioned works, considering CSI imperfection improves performance in terms of various metrics as demonstrated by the reported simulation results, with the cost of design complexity. A specific type of wireless system may have complexity limitation where implementing such a design may degrade the overall performance. In addition, certain wireless systems' estimation errors may be minimal, and in these cases CSI imperfection becomes negligible. Therefore, evaluating the performance gain achieved by considering CSI imperfection due to channel estimation error is of great importance.

The aim of this paper is to evaluate the ergodic mutual information of OFDMA-based SDF networks when the CSI is treated as inaccurate, and a priori knowledge of the error statistics is available. The performance gain is measured in terms of a common metric, the ergodic mutual information, and the amount of channel estimation error is represented by the error statistics. The importance of the analytical results lies in the following:

- Given the estimation error statistics, the ergodic mutual information achieved by accounting for CSI inaccuracy can be evaluated. Thus, system designers can decide if considering CSI inaccuracy is worth the incurred computational complexity.
- Allocating resources (i.e., rate, power, subcarriers, and relay stations pairing) in OFDMA cooperative networks based on the proposed approach achieves a network ergodic mutual information close to the one that can be achieved if perfect CSI is available. In particular, a resource allocation problem can be formulated to maximize the evaluated ergodic mutual information while satisfying the network constraints [19].
- The considered system model is very general, such that the proposed approach can be applied to either uplink or downlink modes, dedicated or non-dedicated relays. Furthermore, although the evaluated ergodic mutual information is for SDF networks, it can be applied to direct transmission in PMP networks because it is a special case of SDF networks.

The remainder of the paper is organized as follows. Section 2 introduces the system model of the OFDMA cooperative relay network under consideration. The SDF ergodic mutual information is derived in Section 3. The performance improvement is evaluated in Section 4, followed by conclusions in Section 5.

2. System model

Consider a single cell scenario with one base station at the center of the cell, multiple relay stations, and multiple subscriber stations. Subscriber stations that are not transmitting can cooperate with other subscriber stations as relay stations (i.e., non-dedicated relays). There are A sources forming the set $\mathcal{A} = \{s_1, \dots, s_a, \dots, s_A\}$. The available B relays form the set $\mathcal{B} = \{r_1, \dots, r_b, \dots, r_B\}$. The destination is symbolized by d . The subscriber stations share a total of N_{sc} subcarriers available to the cell. The set of subcarriers is denoted by $\mathcal{N} = \{1, \dots, n, \dots, N_{sc}\}$. In

OFDMA networks, a subset \mathcal{N}_a^1 of the network subcarriers is exclusively assigned to a source–relay pair, as shown in Fig. 1. We consider a frequency selective fading channel between any pair of communicating stations.

Although the presentation of the system model focuses on networks in uplink mode, the proposed approach can be used to evaluate the ergodic mutual information of both uplink and downlink modes. In SDF [3], the transmission frame² is divided into two halves. During the first half, the relay r_b and destination d , respectively, receive the following OFDM signals:

$$\mathbf{r}^{ab}[j] = \sqrt{\mathbf{P}^a[j]} \mathbf{H}^{ab}[j] \mathbf{s}[j] + \mathbf{z}^{ab}[j], \quad (1)$$

$$\mathbf{r}^{ad}[j] = \sqrt{\mathbf{P}^a[j]} \mathbf{H}^{ad}[j] \mathbf{s}[j] + \mathbf{z}^{ad}[j], \quad (2)$$

where $\sqrt{\mathbf{P}^a[j]}$ is a diagonal $N_a \times N_a$ matrix, and where $\sqrt{p_n^a[j]}$ for $n \in \mathcal{N}_a$ is the power allocated by an MAC resource allocation scheme to the a th source on the n th subcarrier during the j th slot. $\mathbf{H}^{bd} = \text{diag}\{\mathbf{h}^{bd}[j]\}$ is the diagonal channel matrix that models the channel between the relay r_b and destination d ; $\mathbf{h}^{bd}[j]$ is a vector of $H_n^{bd}[j]$ for $n \in \mathcal{N}_a$, which is the n th subcarrier gain of the r_b to d channel during the j th slot.³ $\mathbf{s}[j]$ denotes the data source symbols. The vectors \mathbf{z}^{ab} and \mathbf{z}^{ad} , respectively, represent the additive noise at the source-to-relay and source-to-destination channels and are modeled as circularly symmetric complex Gaussian $\mathbf{z}^{ab} \sim \mathcal{CN}(\mathbf{0}, (\sigma_z^{ab})^2 \mathbf{I})$ and $\mathbf{z}^{ad} \sim \mathcal{CN}(\mathbf{0}, (\sigma_z^{ad})^2 \mathbf{I})$. In the second half, if the source-to-relay ergodic mutual information is greater than a threshold R , the relay cooperates with the source, and the destination receives the following OFDM signal:

$$\mathbf{r}^{bd} \left[j + \frac{T_f}{2} \right] = \sqrt{\mathbf{P}^b} \left[j + \frac{T_f}{2} \right] \left(\sqrt{\mathbf{P}^a} \left[j + \frac{T_f}{2} \right] \right)^{-1} \times \mathbf{H}^{bd} \left[j + \frac{T_f}{2} \right] \hat{\mathbf{r}}^{ab}[j] + \mathbf{z}^{bd} \left[j + \frac{T_f}{2} \right], \quad (3)$$

where T_f is the frame length and $\sqrt{\mathbf{P}^b[j]}$ is a diagonal matrix of $\sqrt{p_n^b[j]}$, $n \in \mathcal{N}_a$ being the power allocated to the b th relay on the n th subcarrier during the j th slot. $\hat{\mathbf{r}}^{ab}[j]$ denotes the re-encoded signal by r_b . The additive noise vector at the relay-to-destination channel $\mathbf{z}^{bd}[j + \frac{T_f}{2}]$ is modeled as $\mathbf{z}^{bd} \sim \mathcal{CN}(\mathbf{0}, (\sigma_z^{bd})^2 \mathbf{I})$. The destination d combines the received signals ($\mathbf{r}^{ad}[j]$ and $\mathbf{r}^{bd}[j + \frac{T_f}{2}]$) by employing one of the literature available diversity combining schemes [3]. Conversely, if the mutual information of the s_a to r_b link is less than R , s_a continues transmitting to d without the cooperation of r_b . The CSI is updated every OFDMA frame. As mentioned previously, the frame is divided into two subframes. At the beginning of the first half of the frame, a sequence of OFDM symbols is transmitted by the source s_a to the relay r_b and destination

d for channel estimation. In the second subframe, the relay r_b transmits another set of training symbols for the destination d to estimate the channel. In addition, it forwards its estimate $\hat{\mathbf{H}}^{ab}$ of \mathbf{H}^{ab} to the destination d which estimates $\hat{\mathbf{H}}^{bd}$ and $\hat{\mathbf{H}}^{ad}$ to obtain $\hat{\mathbf{h}}^{bd}$ and $\hat{\mathbf{h}}^{ad}$. The slot index (i.e., $[j]$) is dropped for simpler notation. Fig. 2 shows the modeling parameters of an illustrative SDF network.

Note that the channel matrices are diagonals of the subcarrier channel gain vectors, namely \mathbf{h}^{ab} , \mathbf{h}^{ad} and \mathbf{h}^{bd} . Let $\hat{\mathbf{h}}^{ab}$, $\hat{\mathbf{h}}^{ad}$ and $\hat{\mathbf{h}}^{bd}$ be their estimates available at the receiver, respectively. Before the next frame estimates arrive, they are treated as deterministic [20] and their delay and estimation error are modeled by $\tilde{\mathbf{h}}^{ab}$, $\tilde{\mathbf{h}}^{ad}$ and $\tilde{\mathbf{h}}^{bd}$ [21]; delay error represents the time difference between the instant the channel is estimated and the instant it is considered by the channel adaptive scheme, during which the channel may change. Hence, given the channel estimates $\hat{\mathbf{h}}^{ab}$, $\hat{\mathbf{h}}^{ad}$ and $\hat{\mathbf{h}}^{bd}$, the imperfect CSI for the three links (i.e., s_a to r_b , s_a to d and r_b to d) are modeled, respectively, as follows:

$$\tilde{\mathbf{h}}^{ab} = \hat{\mathbf{h}}^{ab} + \tilde{\mathbf{h}}^{ab}; \quad (4)$$

$$\tilde{\mathbf{h}}^{ad} = \hat{\mathbf{h}}^{ad} + \tilde{\mathbf{h}}^{ad}; \quad (5)$$

$$\tilde{\mathbf{h}}^{bd} = \hat{\mathbf{h}}^{bd} + \tilde{\mathbf{h}}^{bd}. \quad (6)$$

$\tilde{\mathbf{h}}^{ab}$, $\tilde{\mathbf{h}}^{ad}$ and $\tilde{\mathbf{h}}^{bd}$ are, respectively, assumed to be $\sim \mathcal{CN}(\hat{\mathbf{h}}^{ab}, \Sigma_{\tilde{\mathbf{h}}^{ab}})$, $\sim \mathcal{CN}(\hat{\mathbf{h}}^{ad}, \Sigma_{\tilde{\mathbf{h}}^{ad}})$ and $\sim \mathcal{CN}(\hat{\mathbf{h}}^{bd}, \Sigma_{\tilde{\mathbf{h}}^{bd}})$, where $\Sigma_{\tilde{\mathbf{h}}^{ab}}$, $\Sigma_{\tilde{\mathbf{h}}^{ad}}$ and $\Sigma_{\tilde{\mathbf{h}}^{bd}}$ are the error covariance matrices [21,22]. We assume that the estimation errors on different subcarriers are independent; hence, the covariance matrices are scalar multiples of the identity matrix. Therefore, $\Sigma_{\tilde{\mathbf{h}}^{ab}} = (\tilde{\sigma}_\epsilon^{ab})^2 \mathbf{I}$, $\Sigma_{\tilde{\mathbf{h}}^{ad}} = (\tilde{\sigma}_\epsilon^{ad})^2 \mathbf{I}$, and $\Sigma_{\tilde{\mathbf{h}}^{bd}} = (\tilde{\sigma}_\epsilon^{bd})^2 \mathbf{I}$, where $(\tilde{\sigma}_\epsilon^{ab})^2$, $(\tilde{\sigma}_\epsilon^{ad})^2$, and $(\tilde{\sigma}_\epsilon^{bd})^2$ are the delay and estimation error variances. Hence, the n th subcarrier⁴ imperfect CSI ($[\hat{\mathbf{h}}^{ab}]_n = \hat{H}_n^{ab}$, $[\hat{\mathbf{h}}^{ad}]_n = \hat{H}_n^{ad}$, and $[\hat{\mathbf{h}}^{bd}]_n = \hat{H}_n^{bd}$) are, respectively, modeled as $\sim \mathcal{CN}(\hat{H}_n^{ab}, (\tilde{\sigma}_\epsilon^{ab})^2)$, $\sim \mathcal{CN}(\hat{H}_n^{ad}, (\tilde{\sigma}_\epsilon^{ad})^2)$ and $\sim \mathcal{CN}(\hat{H}_n^{bd}, (\tilde{\sigma}_\epsilon^{bd})^2)$. Therefore, their squares follow non-central Chi-square probability density functions (PDFs) given by [23] as follows:

$$f_X(x) = \frac{1}{(\tilde{\sigma}_\epsilon^{ad})^2} e^{-\frac{(\hat{H}_n^{ad})^2 + x}{(\tilde{\sigma}_\epsilon^{ad})^2}} \mathcal{I}_0 \left(2\sqrt{\frac{|\hat{H}_n^{ad}|^2 x}{(\tilde{\sigma}_\epsilon^{ad})^4}} \right); \quad (7)$$

$$f_Y(y) = \frac{1}{(\tilde{\sigma}_\epsilon^{ab})^2} e^{-\frac{(\hat{H}_n^{ab})^2 + y}{(\tilde{\sigma}_\epsilon^{ab})^2}} \mathcal{I}_0 \left(2\sqrt{\frac{|\hat{H}_n^{ab}|^2 y}{(\tilde{\sigma}_\epsilon^{ab})^4}} \right); \quad (8)$$

$$f_Z(z) = \frac{1}{(\tilde{\sigma}_\epsilon^{bd})^2} e^{-\frac{(\hat{H}_n^{bd})^2 + z}{(\tilde{\sigma}_\epsilon^{bd})^2}} \mathcal{I}_0 \left(2\sqrt{\frac{|\hat{H}_n^{bd}|^2 z}{(\tilde{\sigma}_\epsilon^{bd})^4}} \right), \quad (9)$$

¹ The cardinality of the subset \mathcal{N}_a is denoted by N_a .

² An OFDMA frame consists of multiple time slots. An OFDMA symbol is transmitted on all assigned subcarriers during the same time slot [5].

³ The superscript ab , bd and ad , respectively, denote the link between a source s_a and a relay r_b , a relay r_b and a source s_a , and a source s_a and the destination d .

⁴ $[\mathbf{x}]_n$ denotes the n th element of vector \mathbf{x} .

$$\tilde{I}_{n,a,b}^{\text{SDF}} = \begin{cases} \frac{1}{2} \log \left(1 + \frac{2p_n^a |\hat{H}_n^{ad}|^2}{(\sigma_z^{ad})^2} \right), & \frac{1}{2} \log \left(1 + \frac{2p_n^a |\hat{H}_n^{ab}|^2}{(\sigma_z^{ab})^2} \right) \leq R \quad (\text{a}) \\ \frac{1}{2} \log \left(1 + \frac{2p_n^a |\hat{H}_n^{ad}|^2}{(\sigma_z^{ad})^2} + \frac{2p_n^b |\hat{H}_n^{bd}|^2}{(\sigma_z^{bd})^2} \right), & \frac{1}{2} \log \left(1 + \frac{2p_n^a |\hat{H}_n^{ab}|^2}{(\sigma_z^{ab})^2} \right) > R \quad (\text{b}) \end{cases}$$

Box 1.

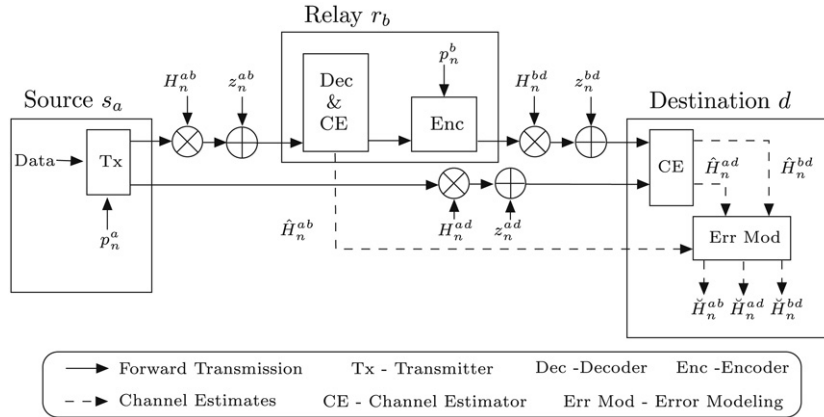


Fig. 2. Illustrative SDF network with the actual, estimated and imperfect CSI on the three links in the uplink mode.

where $\mathfrak{I}_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind. The random variables $|\check{H}_n^{ad}|^2$, $|\check{H}_n^{ab}|^2$ and $|\check{H}_n^{bd}|^2$ are, respectively, denoted by X , Y and Z for simpler notation.

3. SDF ergodic mutual information

The SDF mutual information for deterministic and perfectly known CSI is given by the equation given in Box 1 [3]. When the imperfection of CSI is considered, the channel gains are random variables and the ergodic mutual information becomes a function of them; thus, the equation given in Box 1 becomes

$$E [I_{n,a,b}^{\text{SDF}}] = \begin{cases} E [I_n^{ad} | I_n^{ab}], & \Pr \{I_n^{ab} \leq R\} \quad (\text{a}) \\ E [I_n^{abd} | I_n^{ab}], & \Pr \{I_n^{ab} > R\}, \quad (\text{b}) \end{cases} \quad (10)$$

where $E [I_n^{ad} | I_n^{ab}]$ is the ergodic mutual information of the direct transmission and $E [I_n^{abd} | I_n^{ab}]$ is the cooperation ergodic mutual information, and $\Pr \{I_n^{ab} \leq R\}$ is the probability that the information on the source-to-relay link is less than or equal to a threshold R . With the PDF of $|\check{H}_n^{ad}|^2$, $|\check{H}_n^{ab}|^2$ and $|\check{H}_n^{bd}|^2$, we evaluate the ergodic mutual information of SDF, $E [I_{n,a,b}^{\text{SDF}}]$. In the following, we focus on the mutual information at the subcarrier level; to do so, the subcarrier index n is removed.

3.1. Direct and cooperative transmission probabilities: $\Pr \{I^{ab} \leq R\}$ and $\Pr \{I^{ab} > R\}$

The random variable mutual information between s_a and r_b that is a function of the imperfect CSI random

variable Y is given by [21]

$$I^{ab} = \frac{1}{2} \log_e \left(1 + \frac{2p^a Y}{(\sigma_z^{ab})^2} \right); \quad (11)$$

then,

$$\Pr \{I^{ab} \leq R\} = \int_0^R f_{I^{ab}}(i) di, \quad (12)$$

where $f_{I^{ab}}(i)$ is the PDF of I^{ab} , i.e.,

$$f_{I^{ab}}(i) = 2\zeta e^{2i-\eta-\zeta(e^{2i}-1)} \mathfrak{I}_0 \left(2\sqrt{\eta\zeta(e^{2i}-1)} \right), \quad (13)$$

for $\eta = \frac{|\hat{H}_n^{ab}|^2}{(\hat{\sigma}_e^{ab})^2}$ and $\zeta = \frac{(\sigma_z^{ab})^2}{2p^a(\hat{\sigma}_e^{ab})^2}$. By the transformation $i' = (e^{2i}-1)$, substituting the series representation of $\mathfrak{I}_0(\cdot)$ ([24, 8.447(1)]) into (13) and (13) into (12) gives

$$\Pr \{I^{ab} \leq R\} = \int_0^{e^{2R}-1} e^{-\eta-\zeta i'} \sum_{k=0}^{\infty} \frac{(\eta\zeta i')^k}{(k!)^2} di'. \quad (14)$$

Integrating by parts and rearranging the absolutely convergent series [25], the above integral in (14) evaluates to

$$e^{-\eta} \left[\sum_{m=0}^{\infty} -e^{-\zeta(e^{2R}-1)} \sum_{k=0}^m \frac{\eta^m (\zeta(e^{2R}-1))^k}{m!k!} + \frac{\eta^m}{m!} \right]. \quad (15)$$

By substituting back the values of η and ζ , (15) becomes

$$\Pr \{I^{ab} \leq R\} = e^{-\frac{|\hat{H}_n^{ab}|^2}{(\hat{\sigma}_e^{ab})^2}} \left[\sum_{m=0}^{\infty} \left(-e^{-\left(\frac{(\sigma_z^{ab})^2}{2p^a(\hat{\sigma}_e^{ab})^2}\right)(e^{2R}-1)} \right) \right]$$

$$\times \sum_{k'=0}^m \frac{\left(\frac{|\hat{H}^{ab}|^2}{(\bar{\sigma}_\epsilon^{ab})^2}\right)^m \left(\frac{(\sigma_z^{ab})^2}{2p^a(\bar{\sigma}_\epsilon^{ab})^2}\right) (e^{2R} - 1)^{k'}}{m!k'!} \left(\frac{(\hat{H}^{ab})^m}{(\bar{\sigma}_\epsilon^{ab})^2} \right) + \frac{1}{m!} \quad (16)$$

Hence, $\Pr \{I^{ab} > R\}$ can be simply found from (16) by

$$\Pr \{I^{ab} > R\} = 1 - \Pr \{I^{ab} \leq R\}. \quad (17)$$

3.2. Conditional direct transmission ergodic mutual information $E[I^{ad}|I^{ab}]$

If the mutual information of the s_a to r_b channel is less than the threshold, R , the relay does not cooperate and the source transmits directly to the destination, d . The random variable I^{ad} is a function of the imperfect CSI random variable X (i.e., $|\hat{H}^{ad}|^2$) and is defined as follows:

$$I^{ad} = \frac{1}{2} \log_e \left(1 + \frac{2p^a X}{(\sigma_z^{ad})^2} \right). \quad (18)$$

The PDF of the random variable $Q = \frac{2p^a X}{(\sigma_z^{ad})^2}$ is

$$f_Q(q) = \frac{1}{\Omega_Q^2} e^{-\frac{\alpha_Q^2 + q}{\Omega_Q^2}} \mathcal{J}_0 \left(2\sqrt{\frac{\alpha_Q^2 q}{\Omega_Q^4}} \right), \quad (19)$$

where $\frac{1}{\Omega_Q^2} = \frac{(\sigma_z^{ad})^2}{2p^a(\bar{\sigma}_\epsilon^{ad})^2}$ and $\alpha_Q^2 = \frac{2p^a|\hat{H}^{ad}|^2}{(\sigma_z^{ad})^2}$. By substituting the series representation of $\mathcal{J}_0(\cdot)$ [24, 8.447(1)] in (19), we obtain

$$f_Q(q) = \frac{1}{\Omega_Q^2} e^{-\frac{\alpha_Q^2 + q}{\Omega_Q^2}} \sum_{t=0}^{\infty} \frac{\alpha_Q^{2t} q^t}{\Omega_Q^{4t} (t!)^2}. \quad (20)$$

Given the PDF $f_Q(q)$, the ergodic mutual information for direct communication can be written as

$$E[I^{ad}|I^{ab}] = \int_0^{\infty} \frac{1}{2} \log_e(1+q) f_Q(q) dq \quad (21)$$

$$= \frac{e^{-\frac{\alpha_Q^2}{\Omega_Q^2}}}{2\Omega_Q^2} \sum_{t=0}^{\infty} \frac{\alpha_Q^{2t}}{\Omega_Q^{4t} (t!)^2} \int_0^{\infty} \log_e(1+q) e^{-\frac{q}{\Omega_Q^2}} q^t dq. \quad (22)$$

By [24], 4.222(8), we obtain

$$E[I^{ad}|I^{ab}] = \frac{e^{-\frac{\alpha_Q^2}{\Omega_Q^2}}}{2\Omega_Q^2} \sum_{t=0}^{\infty} \frac{\alpha_Q^{2t} \Omega_Q^{2(t+1)}}{\Omega_Q^{4t} (t!)^2} \sum_{m=0}^t \frac{t!}{(t-m)!} \times \left[\frac{(-1)^{t-m-1}}{\Omega_Q^{2(t-m)}} e^{\frac{1}{\Omega_Q^2}} Ei \left(\frac{-1}{\Omega_Q^2} \right) + \sum_{j=1}^{t-m} \frac{(j-1)!}{-\Omega_Q^{2(t-m-j)}} \right], \quad (23)$$

where $Ei(\cdot)$ is the exponential integral function.

3.3. Conditional cooperative transmission ergodic mutual information $E[I^{abd}|I^{ab}]$

In cooperative networks, diverse assumptions can be made about the network parameters and these will result in different expressions of $E[I^{abd}|I^{ab}]$. This subsection presents $E[I^{abd}|I^{ab}]$ for a general case which is flexible enough to capture the ergodic mutual information of any scenario of SDF networks. In particular, the three links are treated independently such that their power allocations, channel gains and estimation errors statistics are independent. Networks in uplink mode with equal power allocation to source and relay in a pair, and equal estimation error parameters at the relay-to-destination and source-to-destination links are considered a special case. For this special case, the sum of non-central Chi-square distribution can be simplified, which in turn simplifies the general expression of cooperative transmission mutual information.

3.3.1. A general case

Define the random variables $W = \frac{2p^b Z}{(\sigma_z^{bd})^2}$, $S = Q + W$ and

$$I^{abd} = \frac{1}{2} \log_e \left(1 + \frac{2p^a X}{(\sigma_z^{ad})^2} + \frac{2p^b Z}{(\sigma_z^{bd})^2} \right), \quad (24)$$

where the PDF of Z is given by (9). The PDF of Q was found in (19) and the PDF of W is given by

$$f_W(w) = \frac{(\sigma_z^{bd})^2 e^{-\frac{|\hat{H}^{bd}|^2 + (\sigma_z^{bd})^2 w}{2p^b(\bar{\sigma}_\epsilon^{bd})^2}}}{2p^b(\bar{\sigma}_\epsilon^{bd})^2} \times \mathcal{J}_0 \left(2\sqrt{\frac{|\hat{H}^{bd}|^2 (\sigma_z^{bd})^2 w}{2p^b(\bar{\sigma}_\epsilon^{bd})^4}} \right). \quad (25)$$

The distribution of the sum of none-central Chi square random variables is given by [26]

$$f_S(s) = \frac{1}{2\eta_W^2} \left(\frac{s}{\alpha_Q^2} \right)^{1/2} e^{-\frac{s}{2\eta_W^2}} e^{-\frac{1}{2} \left(\frac{\alpha_Q^2}{\eta_Q^2} + \frac{\alpha_W^2}{\eta_W^2} \right)} \times \sum_{i=0}^{\infty} \sum_{l=0}^{\infty} \frac{\Gamma(i+l+1)}{i!l!\Gamma(l+1)} \left(\frac{\sqrt{s}\alpha_W^2\eta_Q^2}{2\alpha_Q\eta_W^4} \right)^l \times \left(\frac{\sqrt{s}(\eta_W^2 - \eta_Q^2)}{\alpha_Q\eta_W^2} \right)^i \mathcal{J}_{i+l+1} \left(\frac{\sqrt{s}\alpha_Q}{\eta_Q^2} \right), \quad (26)$$

where $\alpha_Q^2 = \frac{2p^a|\hat{H}^{ad}|^2}{(\sigma_z^{ad})^2}$, $\eta_Q^2 = \frac{p^a(\bar{\sigma}_\epsilon^{ad})^2}{(\sigma_z^{ad})^2}$, $\alpha_W^2 = \frac{2p^b|\hat{H}^{bd}|^2}{(\sigma_z^{bd})^2}$ and $\eta_W^2 = \frac{p^b(\bar{\sigma}_\epsilon^{bd})^2}{(\sigma_z^{bd})^2}$. Given the distribution of the sum random variable, S , the ergodic mutual information can be evaluated as follows:

$$E[I^{abd}|I^{ab}] = \int_0^{\infty} \frac{1}{2} \log_e(1+s) f_S(s) ds. \quad (27)$$

After writing the Bessel function in its series representation and by using [24, 8.444(8)], the ergodic mutual information in (27) evaluates to the following rearranged series:

$$\begin{aligned}
E [I^{abd}|I^{ab}] &= \frac{1}{4\eta_W^2\alpha_Q} e^{-\frac{1}{2}\left(\frac{\alpha_Q^2}{\eta_Q^2} + \frac{\alpha_W^2}{\eta_W^2}\right)} \\
&\times \sum_{j=0}^{\infty} \sum_{v=0}^j \sum_{o=0}^{j-v} \frac{\Gamma(o+v+1)}{o!v!\Gamma(v+1)} \left(\frac{\alpha_W^2\eta_Q^2}{2\alpha_Q^2\eta_W^4}\right)^v \left(\frac{\eta_W^2 - \eta_Q^2}{\alpha_Q\eta_W^2}\right)^o \\
&\times \frac{\left(\frac{\alpha_Q^2}{\eta_Q^2}\right)^{2j-o-v+1} (2\eta_Q^2)^{j+2}}{(j-o-v)!\Gamma(j+2)} \sum_{m=0}^{j+1} \frac{(j+1)!}{(j+1-m)!} \\
&\times \left[\frac{(-1)^{j-m+1} e^{\frac{1}{2\eta_Q^2}}}{(2\eta_Q^2)^{j-m+1}} \text{Ei}\left(\frac{-1}{2\eta_Q^2}\right) + \sum_{t=1}^{j-m+1} \frac{(t-1)!}{(2\eta_Q^2)^{j-m-t}} \right]. \quad (28)
\end{aligned}$$

3.3.2. A special case

Because the destination, d , is common to both r_b to d and s_a to d links, the estimation and delay errors, and the additive noise variances are equal (i.e., $(\sigma_z^{ad})^2 = (\sigma_z^{bd})^2$ and $(\tilde{\sigma}_\epsilon^{ad})^2 = (\tilde{\sigma}_\epsilon^{bd})^2$). In cooperative networks that have the relay r_b as one of the subscriber stations assisting another subscriber station, the source and the relay can be assumed to be transmitting at equal power, i.e., $p^a = p^b$; thus, $\theta_Q = \theta_W$. Both $f_Q(q)$ and $f_W(w)$ can be mapped to the type-one Bessel function PDF [27], respectively, for $\theta_Q = \frac{(\sigma_z^{ad})^2}{2p^a(\tilde{\sigma}_\epsilon^{ad})^2}$, $\beta_Q^2 = \frac{4\tilde{H}^{ad}|^2(\sigma_z^{ad})^2}{2p^a(\tilde{\sigma}_\epsilon^{ad})^4}$ and $\lambda_Q = 1$, and $\theta_W = \frac{(\sigma_z^{bd})^2}{2p^b(\tilde{\sigma}_\epsilon^{bd})^2}$, $\beta_W^2 = \frac{4\tilde{H}^{bd}|^2(\sigma_z^{bd})^2}{2p^b(\tilde{\sigma}_\epsilon^{bd})^4}$ and $\lambda_W = 1$. By the reproductive property of Bessel function random variables [28], the PDF of S reduces to

$$f_S(s) = \frac{2\theta_S^2}{\beta_S} e^{-\frac{\beta_S^2}{4\theta_S}} s^{\frac{1}{2}} e^{-\theta_S s} \sum_{k=0}^{\infty} \frac{\left(\sqrt{\beta_S^2 s/4}\right)^{2k+1}}{k!(k+1)!}, \quad (29)$$

where $\theta_S = \theta_Q = \theta_W$ (i.e., common variance), and $\beta_S^2 = \beta_Q^2 + \beta_W^2$. Given the PDF of the sum random variable S in (29), the ergodic mutual information is written as

$$\begin{aligned}
E [I^{abd}|I^{ab}] &= \int_0^{\infty} \frac{1}{2} \log_e(1+s) f_S(s) ds. \\
&= \frac{\theta_S^2}{\beta_S} e^{-\frac{\beta_S^2}{4\theta_S}} \sum_{k=0}^{\infty} \frac{\left(\frac{\beta_S}{2}\right)^{2k+1}}{k!(k+1)!} \int_0^{\infty} s^{k+1} e^{-\theta_S s} \log_e(1+s) ds. \quad (30)
\end{aligned}$$

Using [24], 8.444(8), Eq. (30) after rearrangement of terms becomes

$$\begin{aligned}
E [I^{abd}|I^{ab}] &= \frac{e^{-\frac{\beta_S^2}{4\theta_S}}}{\beta_S} \sum_{k=0}^{\infty} \frac{\theta_S^{-k} \left(\frac{\beta_S}{2}\right)^{2k+1}}{k!} \sum_{m=0}^{k+1} \frac{1}{(k-m+1)!} \\
&\times \left[\frac{(-1)^{k-m}}{\left(\frac{1}{\theta_S}\right)^{k-m+1}} e^{\theta_S} \text{Ei}(-\theta_S) + \sum_{l=1}^{k-m+1} \frac{(l-1)!}{\left(\frac{-1}{\theta_S}\right)^{k-m-l+1}} \right]. \quad (31)
\end{aligned}$$

The SDF ergodic mutual information on each subcarrier is obtained by substituting the direct transmission probability (16) and cooperative transmission probability (17), the direct transmission ergodic mutual information (23), and the cooperative transmission ergodic mutual information (28) or (31) in

$$\begin{aligned}
E [I_{n,a,b}^{\text{SDF}}] &= E [I^{ad}|I^{ab}] \Pr \{I^{ab} \leq R\} \\
&\quad + E [I^{abd}|I^{ab}] \Pr \{I^{ab} > R\}. \quad (32)
\end{aligned}$$

Although Eq. (32) is specific to each subcarrier assigned to a subscriber-relay pair, it can be generalized to capture the network ergodic mutual information as follows:

$$E [I_{\text{Net}}^{\text{SDF}}] = \sum_{n=1}^{N_{sc}} \sum_{a=1}^A \sum_{b=1}^B E [I_{n,a,b}^{\text{SDF}}]. \quad (33)$$

4. Performance evaluations

Performance evaluations are presented over two subsections. The first deals with a numerical evaluations that compare the analytically evaluated ergodic mutual information from (32) to that from the equation given in Box I by numerically varying various network parameters. The second provides simulations in which the ergodic mutual information is evaluated for simulated CSI and compared to the analytical results.

4.1. Numerical evaluations

Numerical evaluations illustrate the substantial gain achieved when the CSI imperfection is considered. In addition, they highlight the dependency of the SDF network's mutual information on the CSI accuracy at the three links (i.e., s_a to r_b , s_a to d and r_b to d) as network parameters vary. Consider an SDF network with a subscriber station, a relay station and a destination. Numerical evaluations focus on comparing two scenarios. The *first scenario* represents networks that allocate resources based on the assumption that the received CSI for each subcarrier is perfect, i.e., $|\hat{H}_n^{ab}|^2 = |H_n^{ab}|^2$, $|\hat{H}_n^{ad}|^2 = |H_n^{ad}|^2$ and $|\hat{H}_n^{bd}|^2 = |H_n^{bd}|^2$ in Fig. 2. Under this scenario, the per-subcarrier mutual information expected to be achieved, $\bar{I}_{n,a,b}^{\text{SDF}}$, is given in Box I. The *second scenario* represents more practical networks, in which the estimates of the channels are considered inaccurate, and the inaccuracy is characterized by prior knowledge of the error statistics as in (4)–(6). Based on $|\check{H}_n^{ab}|^2$, $|\check{H}_n^{ad}|^2$ and $|\check{H}_n^{bd}|^2$, Fig. 2, the source achieves the ergodic mutual information $E [I_{n,a,b}^{\text{SDF}}]$ derived in Eq. (32) on each subcarrier assigned to it.

Increasing the signal-to-noise ratio (SNR) on the s_a to r_b link from 3 dB to 23 dB increases the source-to-relay mutual information, represented by the horizontal axis in Fig. 3. Thus, s_a switches from direct transmission mode to the cooperative mode at the threshold, R , as shown in Fig. 3 for $(\tilde{\sigma}_\epsilon^{ad})^2 = (\tilde{\sigma}_\epsilon^{bd})^2 = 0.1$ and equal SNR of 10 dB on the s_a to d and r_b to d links. The cooperation decision is based on the deterministic (i.e., \bar{I}_n^{ab})

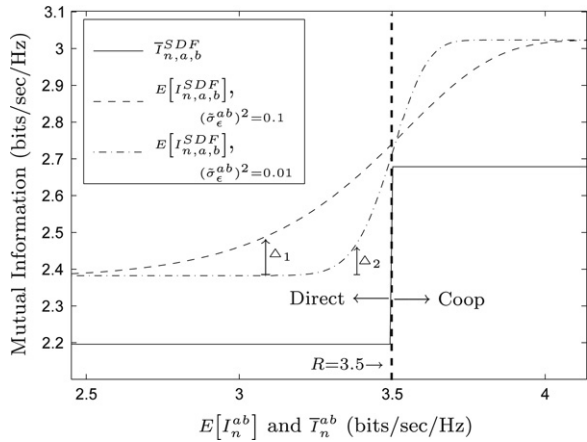


Fig. 3. Change in mutual information as the direct (i.e., s_a to r_b) mutual information increases.

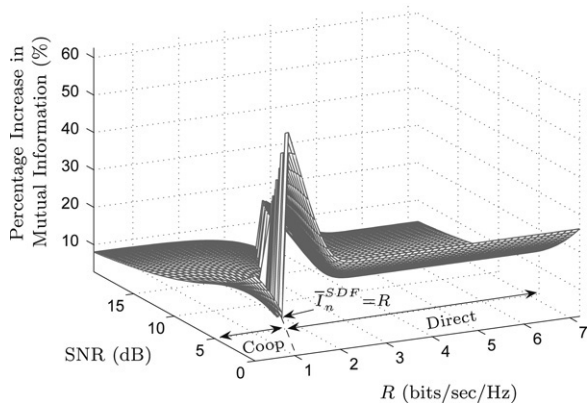


Fig. 4. Percentage increase in mutual information when CSI imperfection is taken into account in both cooperative (Coop) and direct (Direct) transmission modes.

and ergodic (i.e., $E[I_n^{ab}]$) mutual information on the s_a to r_b link in the first and second scenarios, respectively. Thus, considering an inaccurate channel estimate as a perfect one leads to the wrong decision, as is evident from Fig. 3. In particular, because of the estimation and delay errors, the first scenario subscriber stations do not cooperate, although the actual channel condition is at a cooperation permissible level. On the other hand, the second scenario networks consider the error and take advantage of the channel gain increase, which translates to higher mutual information, as indicated by Δ_1 and Δ_2 in Fig. 3. It is obvious that greater mutual information can be achieved by the second scenario networks in either cooperative or direct transmission modes.

Fig. 4 shows the percentage increase in the mutual information achieved, $\frac{E[I_{n,a,b}^{SDF}] - I_{n,a,b}^{SDF}}{I_{n,a,b}^{SDF}} \%$, when the CSI uncertainty is considered at the receiver as the SNR on the three links varies and the threshold R increases for equal error variances, $(\tilde{\sigma}_\epsilon^{ab})^2 = (\tilde{\sigma}_\epsilon^{ad})^2 = (\tilde{\sigma}_\epsilon^{bd})^2 = 0.01$. Although the estimation and delay error is more significant at high SNR compared to low SNR [8], the percentage increase in mutual information is more pronounced at low SNR than

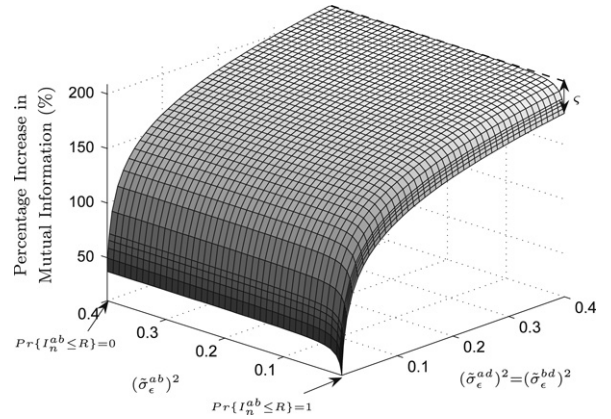


Fig. 5. Effect of CSI inaccuracy on the ergodic mutual information.

at high SNR. This observation is explained as follows: the large gain achieved by considering the second scenario relative to the mutual information of the first scenario at high SNR is less than the small gain achieved by considering the second scenario relative to the mutual information of the first scenario at low SNR. The sharp increase seen in Fig. 4 is due to the difference in transmission modes of each scenario. In other words, the first scenario r_b switches to direct transmission mode based on the poor estimate $|\hat{H}_n^{ab}|^2$, while the second scenario r_b remains in the cooperation mode based on a better estimate, $|\check{H}_n^{ab}|^2$, than $|\hat{H}_n^{ab}|^2$.

To investigate the impact of the three links' CSI imperfection on the ergodic mutual information, Fig. 5 shows the ergodic mutual information gain achieved in the second scenario relative to first scenario as the three estimation errors' variances increase for a constant SNR of 10 dB on the three links. Since the estimation and delay errors at the relay affect the transmission mode decision (i.e., cooperative vs. direct), as $(\tilde{\sigma}_\epsilon^{ab})^2$ increases, the cooperative transmission probability increases, resulting in a gain denoted by ζ over the direct transmission. The increase in $(\tilde{\sigma}_\epsilon^{ad})^2$ and $(\tilde{\sigma}_\epsilon^{bd})^2$ affects both cooperative and direct transmissions' ergodic mutual information; hence, the second scenario achieves larger gain over the first scenario as the estimation and delay errors increase.

4.2. Simulations

Whereas the previous numerical evaluations focused on evaluating the gain achieved by considering the CSI imperfection, the following simulations demonstrate that, with the knowledge of estimation error statistics and CSI estimates, channel adaptive schemes can achieve an ergodic mutual information close to that which would be attained if the perfect CSI were made available through adopting the presented analytical results. Because networks that employ SDF operate in either direct or cooperative modes, we study each mode separately. A frequency selective Rayleigh distributed channel is modeled via a six taps channel model. The number of subcarriers is simulated to be 128 in all simulations. The statistical properties of the perfect, estimated and imperfect CSI for the three links are detailed in Section 2.

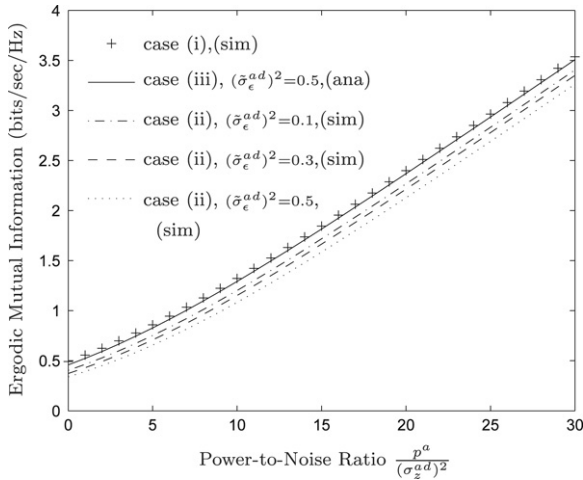


Fig. 6. Ergodic mutual information of the direct mode.

The following discussion compares the ergodic mutual information attained for direct and cooperative modes over a range of power-to-noise ratio for the following cases:

- case (i) The CSI is simulated to be perfectly known (i.e., H^{bd} and H^{ad}) and the ergodic mutual information is evaluated over 1000 channel realizations.
- case (ii) The estimates of CSI (i.e., \hat{H}^{bd} and \hat{H}^{ad}) are the only knowns while the estimation error variances (i.e., $(\tilde{\sigma}_\epsilon^{ad})^2$ and $(\tilde{\sigma}_\epsilon^{bd})^2$) are simulated to be 0.1, 0.3 or 0.5. The ergodic mutual information is evaluated over 1000 channel realizations.
- case (iii) The estimates of CSI (i.e., \hat{H}^{bd} and \hat{H}^{ad}) are known and the estimation error is taken into consideration. The worst estimation error variances of (ii) is considered (i.e., $(\tilde{\sigma}_\epsilon^{ad})^2 = (\tilde{\sigma}_\epsilon^{bd})^2 = 0.5$) where the ergodic mutual information is evaluated analytically by (23) for the direct mode and by (31) or (28) for the cooperative mode.

Note that the source-to-relay mutual information I^{ab} in (10) in SDF networks only determines the probability of direct and cooperative and not the ergodic mutual information in either of the modes; thus, it is not considered in the following simulations.

Figs. 6 and 7 show the ergodic mutual information of the direct and cooperative modes, respectively. In both figures, (sim) and (ana) denote simulated and analytically evaluated ergodic mutual information for each case, respectively. It can be clearly observed that the ergodic mutual information (dashed, dotted and dashed-dotted lines) decreases with the channel estimation mismatch increase that is represented by the estimation error variance increase from 0.1 to 0.5 when the estimation error is ignored (case (ii)). However, with the analytically evaluated expression and the knowledge of the channel estimates and error statistics, the ergodic mutual information (solid line, case (iii)) is close to the one attained when the perfect CSI is available (marked by +, case (i)).

Networks with a dedicated relay station have a high channel gain on the relay-to-base station link because

Fig. 7. Ergodic mutual information of the cooperative mode.

the relay station is fixed and its position is optimized by network planners. Therefore, the channel can be tracked easily and the estimation performance improves. In addition, dedicated relay stations have more relaxed power constraints than non-dedicated ones that are often battery powered. This scenario is simulated by setting $\frac{p^b}{(\sigma_\epsilon^{bd})^2}$ to a high value (e.g., 20 dB) and the estimation error variance to a low value (e.g., $(\tilde{\sigma}_\epsilon^{bd})^2 = 0.01$). Fig. 8 shows the ergodic mutual information of this scenario. It demonstrates that the estimation error effect is larger at high SNR than at low SNR, which concurs with an observation made earlier. Using similar plots, network designers can answer questions about the significance of the CSI imperfection effect in the power operation region of their designed transmitters. As in Figs. 6 and 7, Fig. 8 shows that the analytically evaluated ergodic mutual information captures the ergodic mutual information that can be achieved if the perfect CSI is made available. Thus, adopting the proposed approach in channel adaptive schemes greatly reduces the performance degradation caused by channel estimation inaccuracy. An example of utilizing the analytical results in resource allocation for OFDMA networks is presented in [19].

5. Conclusions

We have proposed an approach to investigate the ergodic mutual information of OFDMA-based SDF cooperative relay networks with imperfect CSI. A quantitative measure of the gain in ergodic mutual information attained by considering the CSI imperfection was presented. Numerical evaluations show that, by considering the CSI imperfection, the ergodic mutual information can be increased over a large range of SNR; however, the relative increase is more tangible at low SNR than that at high SNR. Furthermore, channel adaptive schemes can achieve an ergodic mutual information close to the one achieved if perfect estimates of CSI are available. Therefore, until designed resource allocation schemes consider channel estimates and delay errors, they remain far from being practically im-

