

# Impact of Imperfect Channel State Information on Cooperative Communications

Mohamad Khattar Awad<sup>†</sup>, Veluppillai Mahinthan<sup>‡</sup>, Jon W. Mark, and Xuemin (Sherman) Shen

<sup>†</sup>American University of Kuwait, Salmiya, Kuwait

<sup>‡</sup>Research in Motion, Waterloo, Canada

Centre of Wireless Communications, Department of Electrical and Computer Engineering

University of Waterloo, Waterloo, Ontario N2L 3G1, Canada

Email: <sup>†</sup>mohamad@ieee.org, <sup>‡</sup>mahi@ieee.org, {jwmark, xshen}@bbcr.uwaterloo.ca

**Abstract**—Although cooperative diversity (CD) systems are widely considered in recent wireless communications standards, little research has been undertaken on their performance under practical conditions. In this study, we study the impact of the channel estimation error on the performance of the multilevel quadrature amplitude modulation (M-QAM) based regenerate-and-forward CD systems. In addition, we investigate the effect of user mobility on channel estimation. After a pilot symbol assisted modulation is proposed for CD systems, the analytical BER performance of the proposed CD system is derived and results are verified via simulations. Numerical results demonstrate that estimation error degrades the performance of CD systems. Furthermore, the inter-user channel estimation error has greater impact on the performance of CD systems than that of the user-to-base station channel. Moreover, the choice of pilot symbol insertion period is crucial and it trades effective data rate for BER performance.

## I. INTRODUCTION

Cooperative diversity (CD) emulates multiple input and multiple output transmissions when the transmitting terminals cannot house multiple antennas. User cooperation not only creates a virtual MIMO transmission, but may also improve system performance in the presence of channel fading, because the separation between cooperating users is usually much larger than that between transmitting antennas in a single terminal. CD gains have the effect of improving power consumption and transmission rates, and hence spectral efficiency [1], [2]. Moreover, cooperation can improve network performance by choosing to cooperate only when network conditions permit, i.e., battery life and channel condition [3]. Cooperation can be used in many situations, e.g., in cognitive radio communications [4]. Another technology known for its high spectral efficiency is multilevel quadrature amplitude modulation (M-QAM). These features describe the motivation behind combining both technologies in the considered system.

Coherent CD systems require perfect channel state information (CSI) in terms of channel coefficients. On the other hand, non-coherent CD systems do not require knowledge of CSI to demodulate the received signals at the expense of error performance degradation [5]–[7]. For both coherent and non-coherent CD systems, power is appropriately allocated among the source and the partners based on the CSI, e.g., the variance of the channel coefficient [8]. Perfect CSI is

normally assumed, but in practice, CSI must be estimated at the receiver and fed back to the transmitter. CSI estimation introduces estimation error and processing delay [9], so that in practical systems, CSI values used at the transmitter are imperfect. These issues underscore the importance of studying the effects of channel estimation error on the performance of quadrature signaling-based CD systems.

Although, perfect knowledge of CSI is a common literature assumption with the exception of [10], [11] for amplify-and-forward (AF) CD systems, in practice, CSI is subject to estimation error. The authors in [12] and [13] have investigated the effect of channel estimation error on the performance of AF systems. Despite the difference in the evaluated performance metrics, symbol error rate in [12] and BER in [13], both conclude that noise in channel estimation significantly affects the performance of CD, especially at high Doppler frequency. [14] studies the performance of regenerate-and-forward (RF) for BPSK over non-symmetric Nakagami fading channels. In particular, [14] provides a bit error rate (BER) expression that can be used in evaluating cooperative network performance in specific scenarios. The outage and capacity performance metrics for RF are evaluated in [15]. Expressions in [14] and [15] were derived under the assumption of perfect knowledge of the channel. Assuming imperfect CSI, [16] presents an approximate receiver for RF cooperative networks that employ binary modulation. The author analyzes the performance of the proposed receiver in terms of average BER. According to [16], a 5% decrease in correlation coefficient between the true and estimated channels introduces a degradation by an order of magnitude in average BER. It would seem, therefore, that inaccuracy of channel estimation would have significant impact on the performance of the considered system. Thus, further investigations are needed in order to analytically analyze the BER performance of quadrature-based regenerate and forward CD systems, considering CSI inaccuracy.

The following are the main contributions of this work: (a) proposal of a Pilot Symbol Assisted Modulation (PSAM) scheme to facilitate CSI estimation in Section IIB, and (b) evaluation of system performance in Section IV under the imperfect CSI regime introduced by the estimation error

The remainder of the paper is organized as follows. Sec. II

describes the system model and defines the problem. The performance analysis of the proposed scheme is given in Sec. III. Numerical results are presented in Sec. IV. Finally, concluding remarks are made in Sec. V.

## II. SYSTEM DESCRIPTION

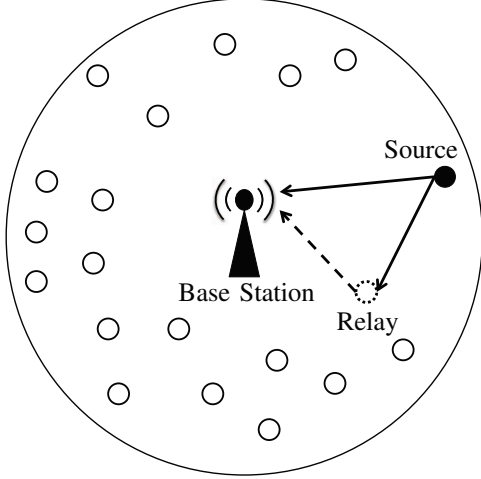


Fig. 1. Cellular network with users cooperation

We consider a wireless cellular network where the base station (BS) of a radio cell supports  $U$  mobile users as shown in Fig. 1. Each user is capable of cooperating with another user, *i.e.*, cooperation between two active users. A cooperating partner for a user is pre-selected by a partner selection algorithm. The partner forwards the frame if it was correctly received. The BS and the mobile devices each has a single antenna. The uplink signals (transmitted by the sender and relayed by the partner) are combined at the BS using maximal ratio combining (MRC). The cooperative diversity scheme thus emulates a “two inputs one output” (2I1O) situation. The inter-user and user-to-BS channels are assumed to exhibit time selective and non-frequency selective Rayleigh fading and independent of each other. The rate of change of CSI over a symbol duration is negligible. Moreover, the receivers are assumed to have proper carrier and symbol synchronization.

### A. Cooperative Diversity Scheme

Consider a cooperative diversity scheme based on  $M$ -QAM signaling in which a user (the sender) cooperates with another user (the partner) to transmit signals in the uplink of an infrastructure based network. Cooperating users transmit their own and their partners’ signals simultaneously using quadrature signaling. In quadrature signaling, we assign the in-phase channels at both users to user-1, *i.e.*,  $I_1$ ,  $I_2$  and  $I_{1,2}$ , and the quadrature channels to user-2, *i.e.*,  $Q_1$ ,  $Q_2$  and  $Q_{1,2}$ , [17].

Fig. 2 shows the frames of symbols broadcasted by user-1 on  $I_1$  and  $I_{1,2}$  to the BS and user-2, respectively. User-2, then relays the information in the next frame to the BS on

$I_2$ . For example, user-1 broadcast the solid black frame on  $I_1$  and  $I_{1,2}$ . Then, user-2 transmits the received solid black frame on  $I_2$  to the BS. The same applies when user-2 broadcasts its frames on  $Q_2$  and  $Q_{1,2}$  which are relayed by user-1 on  $Q_1$ . The signals transmitted by the sender and relayed by the partner are combined at the BS receiver using MRC. Fig. 2 also shows the constellation plot with the in-phase and quadrature channels of each user; the inter-user constellation plot is omitted due to space limitation.

Since user-1 and user-2 use the in-phase and the quadrature components of the  $M$ -QAM modulation, respectively, each user equivalently employs  $\sqrt{M}$ -pulse amplitude modulation (PAM). In the uplink receiver of both users, the in-phase and quadrature components are demodulated separately and the detected and regenerated partner’s frame is forwarded to the BS if the frame is detected error free.

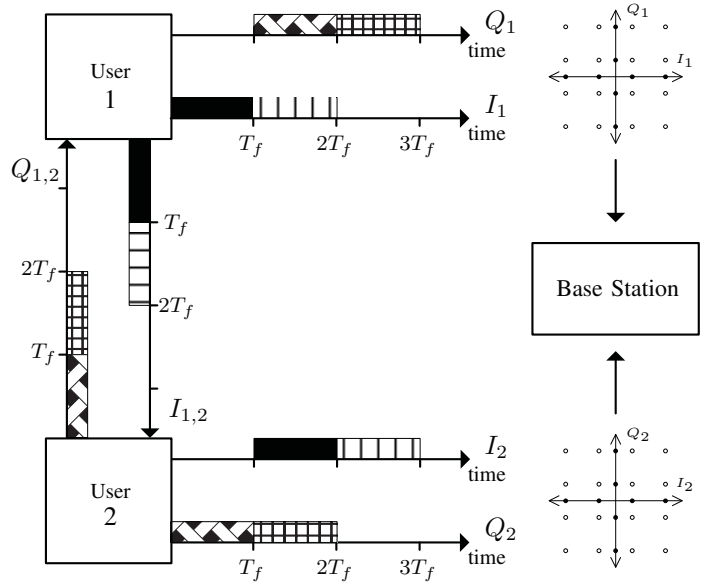


Fig. 2. Illustrative CD network with a cooperating pair (user-1 and user-2), and a base station showing the time line of frames sourcing and relaying on each of the users’ channels.

Let the baseband equivalent received signals from user- $i$ ,  $i = 1, 2$ , at the BS be denoted as  $r_{i,B}(t)$ . Similarly, user- $i$ ’s uplink signal received by user- $j$ ,  $j = 1, 2$  and  $i \neq j$ , is denoted as  $r_{i,j}(t)$ . From the system model,  $r_{i,B}(t)$  and  $r_{i,j}(t)$  can be written as

$$r_{i,B}(t) = h_{i,B}(t)s_i(t) + \eta_{i,B}(t) \quad (1)$$

$$r_{i,j}(t) = h_{i,j}(t)s_i(t) + \eta_{i,j}(t), \quad (2)$$

where the channel coefficient between user  $i$  and the BS is denoted by  $h_{i,B}(t)$  and that from user  $i$  to user  $j$  by  $h_{i,j}(t)$ . The inter-user channel is assumed symmetric, *i.e.*,  $h_{i,j}(t) = h_{j,i}(t)$ . The processes  $\eta_{i,B}(t)$  and  $\eta_{i,j}(t)$  are additive noise at the respective receivers and assumed to be zero mean circularly symmetric, complex Gaussian distribution with variance  $N_0/2$  per dimension, where  $N_0$  is the one-sided power spectral density of white Gaussian noise.  $s_i(t)$ ,

chosen from an M-QAM signal constellation, is the transmitted signal from *user i*. Therefore,  $s_1(t)$  and  $s_2(t)$  can be written as  $s_1(t) = \sqrt{\log_2 M \cdot E_{b1}/2}(a_1(t) + i\bar{a}_2(t - T_f))$  and  $s_2(t) = \sqrt{\log_2 M \cdot E_{b2}/2}(\bar{a}_1(t - T_f) + ia_2(t))$ , respectively, where  $i$  is the unit complex number,  $T_f$  is the frame duration and  $E_{bi}$  is the energy spent per bit using non-cooperative diversity (NCD) transmission at user  $i$ . To have a power consumption equal to that of an NCD system, the CD system equally shares the power in transmitting the user's and the partner's information bits at each mobile device.  $a_i(t)$  is the I-PAM information symbol of *user i*.  $\bar{a}_i(\cdot)$  is the corresponding reproduced symbol at the partner (*user j*), given by

$$\bar{a}_i(t) = \arg \min_{\bar{a}_1 \in S_I} \left( \Re\{\hat{h}_{j,i}^*(t)r_{j,i}(t)\} \right) \quad (3)$$

where  $\hat{h}_{j,i}$  is the estimate of  $h_{j,i}$  and  $(\cdot)^*$  denotes the complex conjugate. At the BS, if each user relays the partner's information, the signals received from both user channels are combined using MRC and decoded using the maximum likelihood (ML) rule. The decoded symbols are given by

$$\begin{aligned} \tilde{a}_1 = \arg \min_{\tilde{a}_1 \in S_I} & \left( \Re\{\hat{h}_{1,B}^*(t - T_f)r_{1,B}(t - T_f) \right. \\ & \left. + \hat{h}_{2,B}^*(t)r_{2,B}(t)\} - (|\hat{h}_{1,B}(t - T_f)|^2 + |\hat{h}_{2,B}(t)|^2)\tilde{a}_1 \right) \end{aligned}$$

and

$$\begin{aligned} \tilde{a}_2 = \arg \min_{\tilde{a}_2 \in S_I} & \left( \Im\{\hat{h}_{2,B}^*(t - T_f)r_{2,B}(t - T_f) \right. \\ & \left. + \hat{h}_{1,B}^*(t)r_{1,B}(t)\} - (|\hat{h}_{1,B}(t)|^2 + |\hat{h}_{2,B}(t - T_f)|^2)\tilde{a}_2 \right), \end{aligned} \quad (4)$$

where  $\Re\{Z\}$  and  $\Im\{Z\}$  denote the real and the imaginary parts of the complex number  $Z$ , respectively.  $S_I$  is the set of I-PAM information symbols.

### B. Pilot Symbol Assisted Modulation

The inaccuracy of channel information degrades the performance of CD systems by affecting demodulation, power allocation and partner selection. It is extremely important to employ a suitable channel estimation mechanism that adapts the system parameters to channel variations. Receivers of the relay and the BS estimate the CSI and demodulate the received signals. The inter-user CSIs are reported to the BS for power allocation and partner selection. The CSIs can be sent through a dedicated feedback channel or placed in the header of the next frame after the estimation. The BS provides the power and partner information to users through the control channel. The accuracy of the CSI depends on the accuracy of CSI estimation, averaging window size of CSI, signal processing delays and feedback delays.

Before presenting the performance analysis, we propose a PSAM-based coherent CD system with sinc interpolation filtering at the receivers to estimate the rapid varying fading channels. The PSAM provides better error performance over non-coherent modulation systems for both quasi-static and time varying fading channels. Simulations later show that the number of used pilot symbols that yields the best performance is a function of the channel variations.

In order to estimate CSI, pilot symbols are periodically inserted in the frame [18]. Each frame is fragmented into  $N$  subframes and each subframe has  $L$  symbols, including one pilot symbol at the beginning of each subframe. The estimates of CSI, such as  $\hat{h}_{i,B}$  and  $\hat{h}_{i,j}$ , are required for coherently detecting data symbols at the partner and the BS, respectively. The receivers of the relay and the destination estimate the CSI and demodulate the received signals.

Without loss of generality, consider inter-user channel estimation. The matched filter output of the received signal for the  $n^{\text{th}}$  subframe's pilot symbol,  $r_{i,j}((n-1)L+1)$ , can be written as

$$\begin{aligned} r_{i,j}((n-1)L+1) &= h_{i,j}((n-1)L+1) \\ &\times S_p((n-1)L+1) + \eta_{i,j}((n-1)L+1) \end{aligned} \quad (5)$$

where  $L$  denotes the length of a sub-frame. The fading channel coefficient can be extracted by dividing the received signal,  $r_{i,j}((n-1)L+1)$ , by the known pilot symbol  $S_p((n-1)L+1)$ . It can be written as

$$\begin{aligned} \tilde{h}_{i,j}((n-1)L+1) &= \frac{r_{i,j}((n-1)L+1)}{S_p((n-1)L+1)} \\ &= h_{i,j}((n-1)L+1) + \frac{\eta_{i,j}((n-1)L+1)}{S_p((n-1)L+1)}. \end{aligned} \quad (6)$$

Since the channel is time varying, the extracted fading channel coefficient,  $\tilde{h}_{i,j}((n-1)L+1)$ , in the pilot symbol duration is not the same as that of the data symbol's duration. In addition, the received signal contains additive white Gaussian noise. Thus,  $\tilde{h}_{i,j}((n-1)L+1)$  is not a perfect estimate of  $h_{i,j}((n-1)L+1)$ . To overcome this problem, a number of filtering methods, including Wiener filter interpolation, Gaussian interpolation and sinc interpolation have been proposed in the literature. In this paper, we use a sinc interpolation filter with  $\omega$  taps which yields performance similar to the optimal Wiener interpolation.

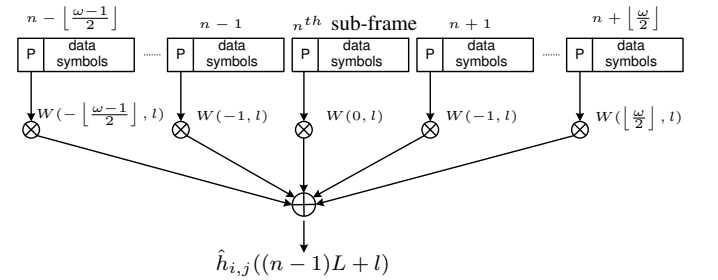


Fig. 3. Fading channel estimator using PSAM

The sinc interpolation filter estimates the CSI for the  $l^{\text{th}}$  symbol in the  $n^{\text{th}}$  subframe from the  $\omega$  nearest pilot symbols. The interpolator uses  $\lfloor(\omega-1)/2\rfloor$  pilot symbols from the previous subframes, the pilot symbol from the current subframe and the pilot symbols from the  $\lfloor\omega/2\rfloor$  subsequent subframes (as shown in Fig. 3) to estimate the CSI. Therefore, the estimated fading channel coefficient for the  $l^{\text{th}}$  data symbol of the  $n^{\text{th}}$

subframe can be written as

$$\hat{h}_{i,j}((n-1)L+l) = \sum_{k=-\lfloor(\omega-1)/2\rfloor}^{\lfloor\omega/2\rfloor} W(k,l)\tilde{h}_{i,j}((n+k-1)L+1) \quad (7)$$

where  $W(k,l) = \text{sinc}(\frac{l}{L} - k)$  and  $l = \{2..L\}$  are real interpolation coefficients.

### III. PERFORMANCE ANALYSIS

Based on the BER performance of the single input and single output (SISO) and MRC systems [19], we derive the analytical BER performance of the proposed cooperative diversity system. The BER,  $P_b(N)$ , for M-QAM is given by Eq. (9) of [19]

$$P_b(O) = \frac{1}{L-1} \sum_{l=1}^{L-1} \left( \frac{4}{M} \sum_{q_1=1+\sqrt{M}/2}^{\sqrt{M}} \sum_{q_2=1+\sqrt{M}/2}^{\sqrt{M}} p_{c,l,q_1,q_2,O} \right) \quad (8)$$

where  $p_{c,l,q_1,q_2,O}$  is the conditional BER of the transmitted symbol,  $s_i(nL+l)$  is  $(2q_1-1-\sqrt{M})d+i(2q_2-1-\sqrt{M})d$ , with  $O^{th}$  order receive diversity. The reader is referred to [19] for additional information on the detailed derivation of  $p_{c,l,q_1,q_2,O}$  which is evaluated by:

$$\begin{aligned} p_{c,l,q_1,q_2,O} &= \frac{2}{\log_2 M} \left\{ H_{\sqrt{M},q_1} P_r \left( (\sqrt{M}-2)d, l, q_1, q_2, O \right) \right. \\ &+ \sum_{q=2}^{\sqrt{M}-1} H_{q,q_1} \left[ P_r \left( (2q-\sqrt{M})d, l, q_1, q_2, O \right) \right. \\ &\quad \left. \left. - P_r \left( (2q-2-\sqrt{M})d, l, q_1, q_2, O \right) \right] \right. \\ &\left. + H_{1,q_1} P_r \left( (2q-2-\sqrt{M})d, l, q_1, q_2, O \right) \right\} \quad (9) \end{aligned}$$

where  $H_{\theta,\vartheta}$  is the hamming distance between code words representing for  $\sqrt{M}$ -AM symbols  $(2\theta-1-\sqrt{M})d$  and  $(2\vartheta-1-\sqrt{M})d$ . Also,  $d = \sqrt{(3 \log_2 M)/(2(M-1))}$  and the probability that the decision variable is greater than  $\beta$  is given by

$$P_r(\beta, d, l, q_1, q_2, O) = \frac{\sum_{o=1}^{O-1} \binom{2O-1}{o} \left( \frac{-v_2}{v_1} \right)^o}{\left( 1 - \frac{v_2}{v_1} \right)^{2O-1}} \quad (10)$$

where constants  $v_1$ ,  $v_2$ , and  $v_0$  are evaluated as follows

$$v_1 = v_0 - \sqrt{v_0^2 + \frac{1}{R_{r|l,q_1,q_2} R_{\hat{r}|l,q_1,q_2} - |R_{r\hat{r}|l,q_1,q_2}|^2}} \quad (11)$$

$$v_2 = v_0 + \sqrt{v_0^2 + \frac{1}{R_{r|l,q_1,q_2} R_{\hat{r}|l,q_1,q_2} - |R_{r\hat{r}|l,q_1,q_2}|^2}}, \quad (12)$$

$$\text{and } v_0 = \frac{-\beta R_{\hat{r}|l,q_1,q_2} \frac{1}{2} R_{r\hat{r}|l,q_1,q_2}^* \frac{1}{2} R_{r\hat{r}|l,q_1,q_2}}{R_{r|l,q_1,q_2} R_{\hat{r}|l,q_1,q_2} - |R_{r\hat{r}|l,q_1,q_2}|^2}. \quad (13)$$

The second moment functions  $R_{r|l,q_1,q_2}$ ,  $R_{\hat{r}|l,q_1,q_2}$  and  $R_{r\hat{r}|l,q_1,q_2}$  of the received and estimated signals are shown in the Appendix.

In the proposed system, the received signal at the partner has diversity order of one, i.e.,  $O = 1$ . Given the BER of the SISO system, the frame error probability at the partner can be written as

$$P_f = 1 - (1 - P_b(1))^{NL}. \quad (14)$$

If the received frame at the partner is error free, the partner forwards the frame, and the destination combines the signals received from both the source and the partner using MRC, i.e.,  $O = 2$ . Otherwise, the destination only uses the signal transmitted from the source, i.e.,  $O = 1$ . In other words, the probability of cooperation is the probability of error free frame received by the partner. Therefore, the effective BER of the proposed PSAM based CD system is given as

$$P_{b,coop} = P_f P_b(1) + (1 - P_f) P_b(2). \quad (15)$$

The analytical BER performance of the CD system,  $P_{b,coop}$ , is validated using results obtained by simulation.

### IV. NUMERICAL RESULTS

In this section, we investigate the effect of channel estimation error on the BER performance for different system parameters such as modulation level, user mobility, number of pilot symbols in a frame, inter-user and user-to-destination channel quality. Unless explicitly stated, modulation order  $M = 4$ , pilot symbol  $S_p = 1 + i$ , the normalized Doppler frequency  $F_D T_s = 0.015$ , length of a sub-frame equals 16 symbols, each frame has 15 sub-frames, the number of taps in the sinc interpolation filter is  $\omega = 15$ , and the two uplink channels are the same ( $SNR_{1,D} = SNR_{2,D}$ ).

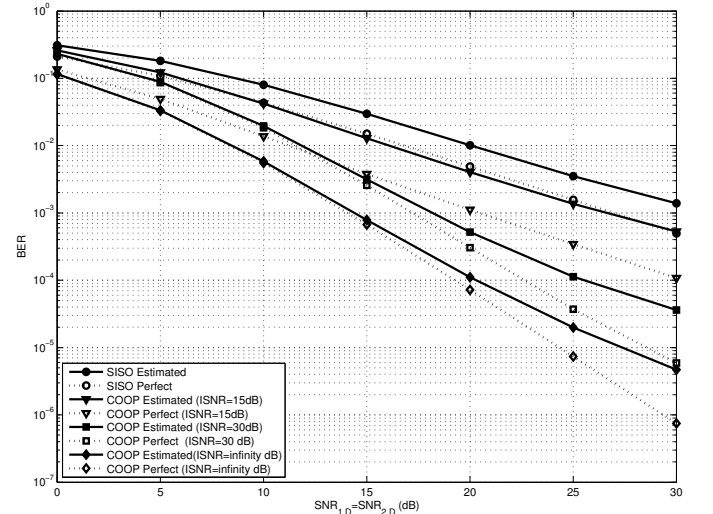


Fig. 4. BER performance of both estimated and perfect CSI ( $SNR_{1,D} = SNR_{2,D}$ ,  $SNR_{1,2} = 30dB$ ,  $L = 16$ ,  $M = 4$ , normalized Doppler=0.015 and  $\omega = 15$ )

First, The BER performance of the channel estimation scheme is compared with the perfect channel assumption in Fig. 4. The performance of CD system with both channel estimation and perfect channel assumption improves as the uplink SNR increases. But, the relative performance degradation

due to channel estimation errors becomes more pronounced when the uplink SNR increases. At the low SNR region, the AWGN seriously corrupts the received signal and the error due to channel estimation is insignificant. On the other hand, the impact of estimation error is significant in the high SNR region compared with AWGN. In addition, the overall CD system performance depends on the inter-user SNR. With high inter-user SNR, the CD system employing channel estimation exhibits less performance degradation compared to the perfect channel case.

The analytical BER is verified by simulation results. Fig. 5 illustrates the BER performance of a 4-QAM based CD system for various inter-user SNRs (ISNR=15dB, 30dB and infinity dB). From the figure, we observe that the analytical results closely match the simulation results. This validates our BER performance analysis based on the analytical approach from the literature for the cooperative system. In addition, the BER performance of the CD system improves with the uplink SNR. The diversity order achieved by the CD system primarily depends on the inter-user channel quality and it performs better when the inter-user SNR increases. The results shown in Fig 6 is the accuracy of the analytical results for various modulation orders ( $M = 4, 16, 64$ ). Similar to the conventional QAM system, the BER performance of the proposed CD system degrades as  $M$  increases for the given SNRs. The higher-order QAM is more sensitive than the lower-order QAM since the minimum Euclidian distance of higher-order QAM signal constellation points is smaller than that of lower-order QAM signal constellation points.

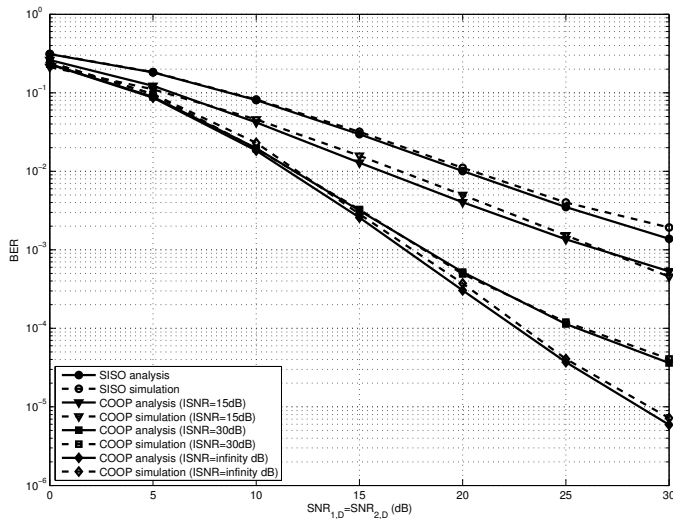


Fig. 5. BER performance of both analysis and simulation ( $SNR_{1,D} = SNR_{2,D}$ ,  $SNR_{1,2} = 30dB$ ,  $L = 16$ ,  $M = 4$ , normalized Doppler=0.015 and  $\omega = 15$ )

Finally, the BER performance versus normalized uplink Doppler frequency is studied in Fig. 7. Basically, increasing the normalized Doppler frequency means increasing the mobile user speed. Thus, the fading channel between the transmitter and the receiver varies rapidly. The channel have

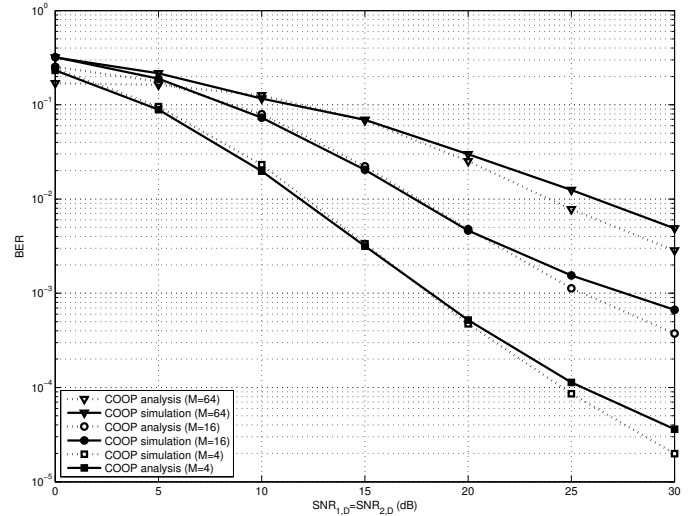


Fig. 6. BER performance for various modulation level ( $SNR_{1,D} = SNR_{2,D}$ ,  $SNR_{1,2} = 30dB$ ,  $L = 16$ , normalized Doppler=0.015 and  $\omega = 15$ )

to be tracked at a high resolution by the estimator. For a finite number of pilot symbols in a frame, the estimator cannot track the channel closely after some limit. It can be accomplished by inserting more pilot symbols in a frame (or reducing the size of a sub-frame) as shown in Fig. 7 for  $L = 16, 12, 8, 4$ . But, increasing the number of pilot symbols reduces the effective data rate. So, there is a tradeoff between estimation performance and effective data rate. The number of pilot symbols to be used to yield the best tradeoff is a function of the channel variations.

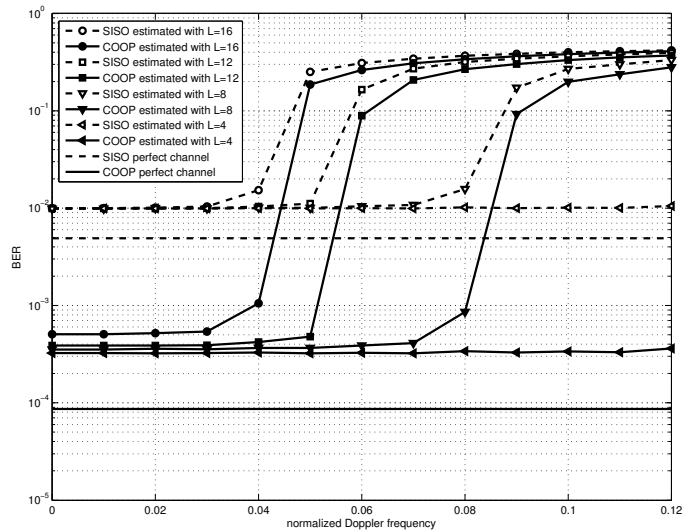


Fig. 7. BER performance versus normalized Doppler of the uplink channels for various sub-frame size ( $SNR_{1,D} = SNR_{2,D}$ ,  $SNR_{1,2} = 30dB$ ,  $M = 4$ , normalized Doppler frequency of inter-user channel=0.015 and  $\omega = 15$ )

## V. CONCLUSION

In this paper, we have analytically analyzed the performance of M-QAM based CD system and proposed a PSAM based channel estimation technique for CD system employing M-QAM. We show that analytical works reported in the literature for conventional and MIMO systems can be extended for cooperative systems with symmetric uplink channels. From the numerical results, it is seen that the inter-user channel estimation error plays an important role in the performance of CD systems. In addition, choosing a pilot symbol insertion period is crucial and it trades off with the effective data rate and BER performance.

It should be noted that this study has been primarily concerned with the BER performance of symmetric uplink channels. Currently, we are investigating the BER performance of the asymmetric time-selective channel towards the destination and optimizing the parameters of the channel estimator and the number of pilot symbols needed for a frame.

## APPENDIX

The second moment-functions of the received and estimated signals for the imperfect channel estimation case are given by [19]

$$R_{r|l,q_1,q_2} = \frac{\Omega \left( (2q_1 - 1 - \sqrt{M})^2 + (2q_2 - 1 - \sqrt{M})^2 \right) d^2}{2} + \frac{N_0}{2}, \quad (16)$$

$$R_{\hat{r}|l,q_1,q_2} = \frac{\Omega}{2} \sum_{k_1=-\lfloor \frac{\omega-1}{2} \rfloor}^{\lfloor \frac{\omega}{2} \rfloor} \sum_{k_2=-\lfloor \frac{\omega-1}{2} \rfloor}^{\lfloor \frac{\omega}{2} \rfloor} f_{k_1}^l f_{k_2}^l \times J_0(2\pi F_D(k_1 - k_2)LT_s) + \sum_{k=-\lfloor \frac{\omega-1}{2} \rfloor}^{\lfloor \frac{\omega}{2} \rfloor} f_k^{2l} \frac{N_0}{2|S_P|^2}, \quad (17)$$

and

$$R_{r\hat{r}|l,q_1,q_2} = \frac{\Omega \left( (2q_1 - 1 - \sqrt{M})d + i(2q_2 - 1 - \sqrt{M}) \right) d}{2} \times \sum_{k=-\lfloor \frac{\omega-1}{2} \rfloor}^{\lfloor \frac{\omega}{2} \rfloor} f_k^l J_0(2\pi F_D(l - k2L)T_s). \quad (18)$$

In the above equations,  $\Omega$  is the average fading power,  $T_s$  is the symbol duration,  $J_0$  is the zeroth-order Bessel function and  $f_k^l$  is the normalized interpolation coefficient that is

$$f_k^l = \frac{W(k,l)}{\sum_{k_1=-\lfloor \frac{\omega-1}{2} \rfloor}^{\lfloor \frac{\omega}{2} \rfloor} W(k_1,l)}. \quad (19)$$

For the case of perfect channel estimation,  $R_{\hat{r}|l,q_1,q_2}$  and  $R_{r\hat{r}|l,q_1,q_2}$  are substituted with

$$R_{\hat{r}|l,q_1,q_2} = \frac{\Omega}{2}, \quad (20)$$

and

$$R_{r\hat{r}|l,q_1,q_2} = \frac{\Omega s_i(nL + 1)}{2}. \quad (21)$$

## REFERENCES

- [1] T. Issariyakul and V. Krishnamurthy, "Amplify-and-forward cooperative diversity wireless networks: Model, analysis, and monotonicity properties," *IEEE/ACM Transactions on Networking*, vol. 17, no. 1, pp. 225–238, Feb. 2009.
- [2] A. Nosratinia, T. Hunter, and A. Hedayat, "Cooperative communication in wireless networks," *IEEE Communications Magazine*, vol. 42, no. 10, pp. 74–80, Oct. 2004.
- [3] P. Liu, Z. Tao, Z. Lin, E. Erkip, and S. Panwar, "Cooperative wireless communications: A cross-layer approach," vol. 13, no. 4, pp. 84–92, Aug. 2006.
- [4] Q. Zhang, J. Jia, and J. Zhang, "Cooperative relay to improve diversity in cognitive radio networks," *IEEE Communications Magazine*, vol. 47, no. 2, pp. 111–117, Feb. 2009.
- [5] D. Chen and J. N. Laneman, "Non-coherent demodulation for cooperative wireless systems," in *Proc. of the IEEE Global Telecommunications Conference (GLOBECOM'04)*, Nov. 2004.
- [6] P. Tarasak, H. Minn, and V. Bhargava, "Differential modulation for two-user cooperative diversity systems," *IEEE Journal on Selected Areas in Communications*, vol. 23, no. 9, pp. 1891–1900, Sep. 2005.
- [7] S. Yiu, R. Schober, and L. Lampe, "Non-coherent distributed space-time block coding," in *Proc. of the IEEE Vehicular Technology Conference (VTC'05)*, Sep. 2005.
- [8] V. Mahinthan, J. W. Mark, and X. Shen, "Partner selection based on optimal power allocation in cooperative diversity systems," *IEEE Transactions on Vehicular Technology*, vol. 57, pp. 511–520, 2008.
- [9] M. K. Awad, X. Shen, and B. Zogheib, "Ergodic mutual information of OFDMA-based selection-decode-and-forward cooperative relay networks with imperfect CSI," *Elsevier Physical Communication*, vol. 2, no. 3, pp. 182–193, Sep. 2009.
- [10] F. Gao, T. Cui, and A. Nallanathan, "On channel estimation and optimal training design for amplify and forward relay networks," *IEEE Transactions on Wireless Communications*, vol. 7, no. 5, pp. 1907–1916, May 2008.
- [11] K. Kim, H. Kim, and H. Park, "OFDM channel estimation for the amplify-and-forward cooperative channel," in *IEEE Vehicular Technology Conference (VTC'07)*, Apr. 2007.
- [12] Y. Wu and M. Patzold, "Performance analysis of cooperative communication systems with imperfect channel estimation," in *Proc. of the IEEE International Conference on Communications (ICC'09)*, Jun. 2009.
- [13] S. Han, S. Ahn, E. Oh, and D. Hong, "Effect of channel-estimation error on BER performance in cooperative transmission," *IEEE Transactions on Vehicular Technology*, vol. 58, no. 4, pp. 2083–2088, May 2009.
- [14] A. Adinoyi and H. Yanikomeroglu, "On the performance of cooperative wireless fixed relays in asymmetric channels," in *Proc. of the IEEE International Conference on Global Communications (GLOBECOM'06)*, Dec. 2006.
- [15] A. Adinoyi, Y. Fan, and H. Yanikomeroglu, "Performance of selection relaying and cooperative diversity," *IEEE Transactions on Wireless Communications*, vol. 8, no. 12, pp. 5790–5795, Aug. 2009.
- [16] R. Annamajjala, "On optimum regenerative relaying with imperfect channel knowledge," *IEEE Transactions on Signal Processing*, vol. 58, no. 3, pp. 1928–1934, Mar. 2010.
- [17] V. Mahinthan, J. W. Mark, and X. Shen, "Performance analysis and power allocation for M-QAM cooperative diversity systems," *IEEE Transactions on Wireless Communications*, vol. 9, no. 3, pp. 1237–1247, Mar. 2010.
- [18] J. K. Cavers, "An analysis of pilot symbol assisted modulation for rayleigh fading channels," *IEEE Transactions on Vehicular Technology*, vol. 40, no. 4, pp. 686–693, Nov. 1991.
- [19] B. Xia and J. Wang, "Effect of channel-estimation error on QAM system with antenna diversity," *IEEE Transactions on Communications*, vol. 53, no. 3, pp. 481–488, Mar. 2005.